GALCIT REPORT NO.________

GUGGENHEIM AERONAUTICAL LABORATORY

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA

FORM AL-5 6M 2-49

AN INVESTIGATION OF THE STRESSES AND DEFLECTIONS OF SWEPT PLATES

Thesis by

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In Partial Fulfillment of the Requirements
for the Degree of
Aeronautical Engineer

California Institute of Technology
Pasadena, California
1949

ACKNOWLEDGEMENT

The author wishes to express appreciation to Mr. Harold M. De Groff for his constant help in the experimental work and his suggestions concerning the analysis, and to Mr. Harold Lurie and Dr. Y. C. Fung for their help in the analysis.

He also wishes to express his gratitude to Professor E. E. Sechler and the members of the GALCIT swept wing research group for their help in carrying out this investigation.

SUMMARY

This report covers one phase of a continuing investigation of the stresses and deflections in swept wings of high solidity. The experimental work consisted of testing a solid plate having the shape of a parallelogram, under bending and torsion, to determine the stress and deflection patterns for angles of sweep up to sixty degrees. The torsion vector at the tip was applied perpendicular to the root.

Under all loadings the area of critical stress is at the root near the trailing edge. Under bending loads the stresses near the trailing edge do not vary with angle of sweep up to forty degrees; at sixty degrees the trailing edge stresses decrease.

Near the leading edge the stress pattern varies sharply with angles of sweep, the stresses near the root becoming negligible at high angles, and the stresses in the outer portion of the span becoming greater.

Under uniform shear and uniformly distributed loading, for all angles of sweep, the area of "end effect" extends to approximately three-quarters of a chord length outboard of a line perpendicular to the axis through the trailing edge root.

This investigation was carried out at the Guggenheim Aeronautical Laboratory of the California Institute of Technology during the academic year 1948-1949.

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I. INTRODUCTION

In the summer of 1947 the Guggenheim Aeronautical Laboratory of the California Institute of Technology (GALCIT) was granted a contract by the U. S. Air Force to investigate the effect of sweep upon the deflection and stress patterns of aircraft wings of high solidity. The investigation is being carried out both theoretically and experimentally and this report is essentially a critical analysis of one phase of the experimental work.

Since little or no published material exists on this subject it was necessary to begin the work with some comparatively elementary studies of the behavior of solid plates having the shape of swept wings and subjected to uniform shear loading, uniformly distributed loading and torsion. By September of 1948 a preliminary investigation on a thin plate had been completed by the GALCIT staff. This work "pointed the way" to the present investigation just as this paper will suggest several points to be considered in further investigation of the problem.

The specimen used in the present tests was a "thick" plate of 24 S-T aluminum alloy ten inches wide by one inch thick and having a length of forty inches between the support and the tip. Four configurations, corresponding to angles of sweep of zero, twenty, forty and sixty degrees, were tested under uniform shear, uniformly distributed, and torsion loadings. Loading was progressive in each of the three types and readings were taken to obtain both deflection and stress at a representative number of points under each type of

load. It was found that the deflections became large at the trailing edge near the tip under all types of loading. The stresses build up rapidly near the root at the trailing edge, for all angles of sweep and all loadings but this condition appears to become more critical with larger sweep angles. For more exact data on the stresses in this region it is recommended that a more complete investigation be made.

This investigation was carried out in the GALCIT structures laboratory under the supervision of Dr. E. E. Sechler, Professor of Aeronautics at the California Institute of Technology. It was done in conjunction with Lt. Comdr. Ralph S. Chandler, U. S. Navy, during the academic year 1948-1949.

II. EQUIPMENT AND PROCEDURE

The test specimen used throughout this investigation was a plate of 24 S-T aluminum alloy, ten inches wide by one inch thick and originally just over six feet long. The test portion was forty inches long and this was maintained constant in the swept configuration by cutting triangular pieces from the free end so as to leave this edge parallel to the support. The dimensions of the specimen in the four configurations of zero, twenty, forty, and sixty degrees of sweep are shown in Figs. 4 to 15.

Standard SR-4 strain rosettes manufactured by the Baldwin-Southwark Co. were attached to the specimen at the points indicated in the above figures. These were connected to a wheatstone bridge circuit from which were taken the strain readings in millivolts. These readings were then converted to principal stresses.

The support for the specimen consisted of a massive steel framework made up of I beams and solid plates. This support is shown in Figs. 1 and 2. It was bolted to the concrete floor and results show that a reasonable degree of rigidity was achieved. Since complete fixity was not possible a survey was made as described below to determine the amount of "sag".

The test specimen was inserted between the two solid plates at the top of the support and surrounded by specially cut spacers. These spacers were used in an effort to obtain a uniform pressure over the fixed end of the specimen. Located beneath the specimen was a large

smooth table.

Deflections under load were obtained by measuring the change in distance, to the nearest thousandth of an inch, between this table and the specimen when the various loads were applied. For this purpose a dial deflection gauge manufactured by the B. C. Ames Company was used. Zeros were obtained before and after loading and it was found that at least three loading cycles were needed in order to stabilize these. Deflection readings were taken at intervals of two and one-half inches axially and at the zero, twenty-five, fifty, seventy-five and hundred per cent chord points. The range of these points was from as near as practicable to the root out to approximately seventy-five per cent of the "semi-span".

Three types of loading were used on each configuration, these being hereafter referred to as uniform shear, uniformly distributed, and torsion. The uniform shear load was applied through a whiffletree arrangement, as shown in Fig. 1, to obtain a uniform distribution along the chord at one hundred per cent semi-span. The uniformly distributed load was applied by distributing shot bags evenly over a foam rubber pad on the surface. The torsion load was applied by attaching a bar to the free end to which were attached two cables. One of these cables ran over an overhead pulley to a tray and the other went directly to a tray as shown in Fig. 2. For the uniform shear load a maximum of six hundred pounds of shot were placed in

the tray suspended from the whiffle-tree. For the uniformly distributed load a maximum of twelve hundred pounds of shot were spread over the specimen. In the torsion loading a maximum of forty-five thousand inch pounds were applied. Under each loading, readings were taken for comparison at loading increments of one-third, two-thirds and maximum. Only the maximum readings were plotted, both of deflection and stress.

The deflection readings as taken were plotted as shown in Figs. 16 to 27. Since the magnitude of the deflection is of interest rather than the direction, the absolute value is plotted without regard to sign, except where a given test has deflections in both directions. Under uniform shear and uniformly distributed loads the deflections are all in the direction of loading. Under torsion the deflection direction reverses, for certain areas, as the sweep angle is increased.

As the results of the plot for a zero sweep angle differed measurably from those derived from theoretical calculations, a survey was made to determine the amount of sag in the support. A lightweight I beam was clamped to the top of the support and its deflection measured when the specimen was loaded. An arch was mounted on the top of the main support and the deflection of the top support plate measured when the load was applied. Finally the sag of the bottom support plate was measured. The combined results of this survey, for sag in the plane perpendicular to the support, are shown in Fig. 36. The sag in the plane parallel to the support was found to be negligible.

Using the corrected values for deflection, cross plots were then made to show the variation in deflection with increased angle of sweep for points on the fifty and seventy-five per cent semi-span lines. Fig. 34 shows this variation for the uniform shear and uniformly distributed loadings and Fig. 35 shows it for the torsion loading.

The orientation and magnitude of the principal stresses at the various strain rosette locations are shown in Figs. 4 to 15. Cross plots were made as shown in Figs. 28 to 30 to show the variation in stress magnitude near the trailing edge for the various sweep angles. Similar plots were made for the stresses near the leading edge as shown in Figs. 31(a) to 33(b). Data for these plots are listed in Tables 1 and 2.

Figs. 31(b) and 33(b) were traced from Figs. 31(a) and 33(a) respectively and then three additional curves were drawn on each one. These curves are representations of the standard engineering formulas for stresses in a simple cantilever beam. For these computations the beams were considered to have fixed roots on a line perpendicular to the plate axis through the trailing edge root. This is shown in each figure.

III. RESULTS AND DISCUSSION

A. STRESSES

A visual picture of the orientation and and magnitude of the principal stresses at the various strain gauge locations is shown in Figs. 4 to 15. It can be seen that the stresses near the trailing edge will be critical for all angles of sweep, particularly in the area near the root. It is in this area that the data are insufficient for a complete analysis. Plots of the variation in maximum stress with distance from the root, near the leading and trailing edges, are shown in Figs. 28 to 33(b).

1. Trailing edge - For the uniform shear load the stresses increase linearly for the outer 80% of the span and then rise very sharply to the root. For the uniformly distributed load the stresses increase approximately parabolically, having almost the same curvature as the engineering formula. Under both types of loading it is noted that the stresses for zero, twenty, and forty degrees of sweep are equal for the outer 85% of the plate. Those for sixty degrees of sweep are measurably less. In both cases use of the standard engineering formula for a cantilever beam gives results which are conservative by ten per cent or more for the outer 85% of the plate. Nearer the root the stresses rise sharply above the formula results for all angles of sweep.

For the torsion load the stresses near the trailing edge increase sharply with sweep. This is due to the manner in which the torsion

load was applied. Since the torsion vector is perpendicular to the root at the tip the plate is subjected to more and more bending as the sweep angle is increased. For each angle the stresses have approximately a constant value for the outer 75% of the plate and then rise sharply to the root.

2. Leading edge - For the uniform shear and uniformly distributed loads the point of maximum stress near the leading edge moves rapidly outboard with increase in angle of sweep as shown in Figs. 31(a) and and 33(a). Inboard of this point of maximum stress the magnitude decreases rapidly, particularly for the higher angles of sweep, and the stresses become negligible at the leading edge root.

For the four angles of sweep investigated, a point of interest worthy of further investigation is noted. Under the uniform shear load, the distance from the root to the point of maximum stress varies linearly as shown in Table 5.

For the torsion loading, the stresses near the leading edge are shown in Fig. 32. On the zero sweep specimen the tensile and compressive stresses are equal and constant for the outer 80% of the span. Near the root the tensile stresses increase while the compressive stresses decrease due to end effect. As the angle of sweep increases from zero to sixty degrees the tensile stresses (on top of the specimen) steadily decrease, while the compressive stresses increase rapidly at the outer end of the span. For the inboard end the compressive stresses reverse this trend, decreasing more and more sharply as the angle of sweep is increased. The innermost point for high compressive

stress moves progressively outboard with increase in angle of sweep, the magnitude of this high stress likewise increasing. These compressive stresses at large angles of sweep are due primarily to bending, rather than torsion, due to the manner of loading. Opposite stresses exist on the bottom of the specimen.

3. End effect - In Figs. 31(b) and 33(b) the curve for the standard engineering formula is different for each angle of sweep. This is due to the fact that only a portion of the plate is considered as a simple cantilever beam. This is the portion outboard of a line drawn through the trailing edge root, perpendicular to the axis of the plate. The theoretical loading in each case is modified from the actual loading as shown in the figures.

For the uniform shear and uniformly distributed loadings Figs. 28, 30, 31(b), and 33(b) show that the area of "end effect", for all angles of sweep, extends outward from the root to a line perpendicular to the plate axis, three-quarters of a chord length outboard of the trailing edge root. Outboard of this line, for all angles of sweep, the theoretical results for the uniformly distributed load agree very well with those obtained experimentally. Near the leading edge for the uniform shear load this is not the case, for the zero sweep angle nor for the sixty. The formula gives conservative results for the former, agrees very well for twenty and forty and then is non-conservative for sixty degrees of sweep.

As noted above under the discussion for the trailing edge, the theoretical results, under uniform shear load, agree very well up to

forty degrees of sweep. At sixty degrees of sweep the disagreement is marked. This leads to the conclusion that for angles of sweep greater than approximately forty-five degrees, the simplifying assumptions of the engineering formula are no longer valid.

B. DEFLECTIONS

For all types of loading the deflections remained in the linear range and the "zeros" measured after removing the loads agreed, within extremely narrow limits, with the initial zeros. For the zero angle of sweep and the uniform shear load, the deflections were slightly greater than those computed from the standard engineering formula for cantilever beams. This is shown in Table 6.

The deflection at the various chord points for fifty and seventy-five per cent of the semi-span under uniform shear and uniformly distributed loads is shown in Fig. 34. The points of maximum deflection occur at the trailing edge under both types of loading and would occur for an angle of sweep between twenty and twenty-five degrees. The deflection of the leading edge decreases rapidly with increase in angle of sweep.

The deflection at the various chord points for fifty and seventy-five per cent of the semi-span under torsion load is shown in Fig. 35. The deflection of the trailing edge increases sharply with increase in angle of sweep. The deflection of the leading edge for small angles of sweep is in the direction of the torque. As the sweep angle increases, the deflection at any given span point decreases to zero and then increases in the opposite direction. It reaches a maximum in this

direction at an angle of sweep of approximately forty-five degrees and then decreases again.

C. ACCURACY

Under all types of loading, when the load was removed the specimen returned to its original position within three thousandths of an inch, an error of less than one-half of one per cent of the maximum deflection. This cannot be said for the return to electrical zero in the wheatstone bridge circuit in all cases. However, the error in return to zero bears no relation to the magnitude of the stress. In the large majority of cases the return is excellent but in a number of instances the return was off by as much as ten per cent of the measured reading. When the readings are converted to principal stresses, some of this error is averaged out. The fairing in of the curves, in Figs. 28 to 33, tends to further average out the error.

When the principal stresses resulting from the one-third maximum load were multiplied by three, the two-thirds by three-halves, and both were compared with those resulting from the maximum load, the maximum error was found to be of the order of five per cent. There was a random direction to this error and it is believed that the variation is due to both inherent lack of accuracy in the electronic equipment and to the inexperience of the operating personnel.

Some estimation of the accuracy of stress measurement is possible from the torsion readings for the case of zero sweep. The deflection readings indicate that the torsion vector was very close to its intended direction. Therefore the compressive and tensile stresses should

be constant in the area which is not subjected to "end effect". Taking from Tables 1 and 2 the last five torsion readings for the zero sweep case, both compressive and tensile, averaging them and then investigating the magnitude of the error leads to the following conclusion: there is an average error of less than plus or minus three per cent and a maximum error of six and a quarter per cent.

As shown in Fig. 36 the sag of the support decreased with increase in angle of sweep. At 75% of the semi-span, for the case of zero sweep, the measurable sag was twenty-four thousandths of an inch or approximately two and one-half per cent of the experimental deflection. At sixty degrees of sweep the measurable sag was slightly greater than one per cent.

Table 6 shows the difference between the deflections as measured for zero sweep angle, and those computed from the standard engineering formula for cantilever beams. The results given by the standard formula were compared with those given by Stevenson's exact formula in Ref. (a). This latter comparison was for points along the centerline of the plate and the difference was negligible. In Stevenson's formula he sets the boundary conditions only at one point, the center of the plate at the root, where he assumes zero deflection and zero slope. In this investigation the plate was clamped along the entire root chord, which leads to more boundary conditions than unknown constants in the formula. In addition these boundary conditions are not known exactly. For this reason the standard engineering formula is believed to be as

nearly exact as any known.

While the error in stress measurement may be as high as six per cent in some isolated cases and the deflection error may vary from three per cent upwards, these errors do not affect the general results stated above. These results are trends and are derived from curves in which the errors are automatically reduced.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

1. Under all types of loading for angle of sweep greater than zero the critical stresses occur at the trailing edge at the root, these stresses increasing sharply over the inner 20% of the span.

2. Under all types of loading the stresses at the leading edge at the root decrease sharply with increase in angle of sweep.

3. Along the leading edge the point of maximum stress for all types of loading moves outboard with increase in angle of sweep.

4. Under uniform shear and uniformly distributed loadings the area of "end effect" for all angles of sweep extends to approximately three-quarters of a chord length outboard of a line perpendicular to the axis through the trailing edge root.

5. Under uniform shear and uniformly distributed loading, use of the standard engineering formulae for stresses in a cantilever beam give good results for angles of sweep up to forty-five degrees if only that portion of the plate not subject to "end effect" is considered.

6. Under uniform shear and uniformly distributed loading the stresses at the trailing edge for the outer 85% of the span do not vary with angle of sweep up to forty degrees. For sixty degrees the stresses decrease measurably.

7. For angles of sweep in excess of forty-five degrees, under uniform shear loading, the simplifying assumptions in the simple beam formulae are no longer valid.

8. The points of maximum deflection under the uniform shear and uniformly distributed loadings occur at the trailing edge for an angle of sweep between twenty and twenty-five degrees.

9. The deflections under uniform shear loading for zero sweep are slightly greater than those given by the standard engineering formula for a cantilever beam.

B. RECOMMENDATIONS

1. The stresses in the area around the trailing edge root should be more thoroughly investigated.

2. For the same amount of time spent, more valuable results could be obtained from taking readings under the maximum loads only.

3. With the results of this experiment in hand, more valuable information could be obtained from fewer strain rosettes which were more advantageously placed.

V. REFERENCE

(a) I. S. Sokolnikoff, "Mathematical Theory of Elasticity", McGraw-Hill Book Company, Inc. 1946 - Page 231.

TABLE 1

Stresses at Ninety Per Cent of Chord

Distance*	Stresses (psi)			
	Uniform Shear Load	Uniformly Distributed Load	Torsion Load	
	+	+	-	+
		$\beta = 0°$		
1.00	12858	11314	14062	4480
5.00	11486	9303	11340	9624
9.00	9855	7054	10554	11512
13.00	8595	5316	11109	10910
17.00	7460	4124	10974	11544
30.00	3267	791	10927	11658
34.00	2025	236	11541	11068
		$\beta = 20°$		
1.20	17408	15486	21985	5497
5.20	12285	10100	17961	8418
9.20	10324	6882	17125	9022
13.20	8726	5080	17148	8952
17.20	7435	3975	17098	8943
30.30	2921	631	16973	8776
34.30	1490	117	17022	8331
		$\beta = 40°$		
3.17	13359	13425	24515	3948
7.17	10979	8375	21497	4879
11.17	9223	6060	20413	5257
15.17	7644	4555	20119	5137
32.17	2010	270	20459	4882
36.17	764	125	17179	3936
		$\beta = 60°$		
2.67	15178	11702	31323	1284
4.67	11471	8383	26394	1902
8.67	9143	5708	23937	2249
12.67	7422	3635	23404	2183
16.67	5677	2467	23016	2253
24.67	3174	817	22328	2038

*Distance is measured in inches from root along chord line.

TABLE 2

Stresses at Ten Per Cent of Chord

Distance*	Stresses (psi)			
	Uniform Shear Load	Uniformly Distributed Load	Torsion Load	
	+	+	-	+
		$\beta = 0^\circ$		
1.00	13053	12550	3358	14619
5.00	11425	9990	9329	11795
9.00	9423	6820	10881	11082
13.00	8555	5428	10346	11846
17.00	7077	3415	10935	10980
26.00	4568	1325	11550	11439
30.00	3198	762	10619	12018
		$\beta = 20^\circ$		
2.30	10350	10148	6776	9700
4.30	11147	10091	10678	8816
8.30	11265	9569	15118	8755
12.30	9722	6331	15788	8996
16.30	8821	4991	16831	8993
20.30	7166	2241	16819	8928
29.30	4323	1185	17269	9246
33.30	2853	362	17203	9397
		$\beta = 40^\circ$		
1.80	4348	4880	183	7577
3.80	6528	6356	3023	5445
5.80	8417	7831	7816	4236
7.80	9747	8498	12210	4219
9.80	10329	8694	15397	4280
13.80	10331	8524	19265	4817
17.80	8697	5446	19171	5308
21.80	4201	4002	19984	5188
25.80	6130	2662	19599	5366
34.80	3105	647	20100	5278

*Distance is measured in inches from root along chord line.

TABLE 2 (Cont'd)

Stresses at Ten Per Cent of Chord

Distance*	Stresses (psi)			
	Uniform Shear Load	Uniformly Distributed Load	Torsion Load	
	+	+	-	+
		β = 60°		
2.30	684	274	176	3527
4.30	1652	863	918	3093
6.30	4640	1783	1811	2807
8.30	3153	2758	3029	2020
10.30	4394	3778	5284	1383
12.30	6321	4830	9036	1220
14.30	7859	6120	13546	1372
16.30	8765	6609	17325	1703
18.30	9133	6508	19821	1813
22.30	8324	4968	22564	2117
26.30	7141	3976	21342	2653
30.30	6052	2407	23208	2309

*Distance is measured in inches from root along chord line.

TABLE 3

Deflections at Fifty Per Cent Semi-span

β °	Deflections (inches)				
	Uniform Shear Load				
	L.E.	25%C	50%C	75%C	T.E.
0	.463	.463	.469	.466	.464
20	.416	.445	.475	.507	.531
40	.260	.323	.382	.448	.518
60	.055	.119	.209	.326	.445
	Uniformly Distributed Load				
0	.412	.414	.414	.413	.413
20	.392	.410	.434	.455	.475
40	.253	.306	.358	.403	.456
60	.061	.114	.176	.263	.357
	Torsion Load				
0	-.300	-.150	.020	.180	.335
20	-.060	.100	.270	.445	.630
40	.070	.240	.440	.690	.950
60	.010	.150	.390	.770	1.190

TABLE 4

Deflections at Seventy-Five Per Cent Semi-span

Deflections (inches)

β °	L.E.	25%C	50%C	75%C	T.E.
			Uniform Shear Load		
0	.953	.956	.956	.956	.953
20	.852	.897	.942	.985	1.023
40	.648	.727	.802	.902	1.001
60	.258	.388	.520	.672	.838
			Uniformly Distributed Load		
0	.751	.751	.751	.751	.751
20	.739	.761	.787	.810	.837
40	.552	.607	.670	.727	.792
60	.220	.316	.398	.501	.606
			Torsion Load		
0	-.460	-.220	.025	.270	.515
20	.135	.380	.640	.920	1.200
40	.495	.790	1.130	1.500	-----
60	.350	.710	1.180	1.600	-----

Table 5

Variation of Maximum Stress Location with Angle of Sweep
Along Ten Per Cent Chord Line

$\beta°$	Distance from root (inches)
0	0
20	6
40	12
60	18

Table 6

Experimental versus Theoretical Deflection of Cantilever Beam

Distance from Root (inches)	Deflections Theoretical	Experimental	Difference	%
32.5	1.046	1.085	.039	3.7
30.0	.917	.956	.039	4.25
27.5	.792	.830	.038	4.8
22.5	.559	.595	.036	6.43
17.5	.356	.385	.029	8.15
12.5	.190	.218	.028	14.7

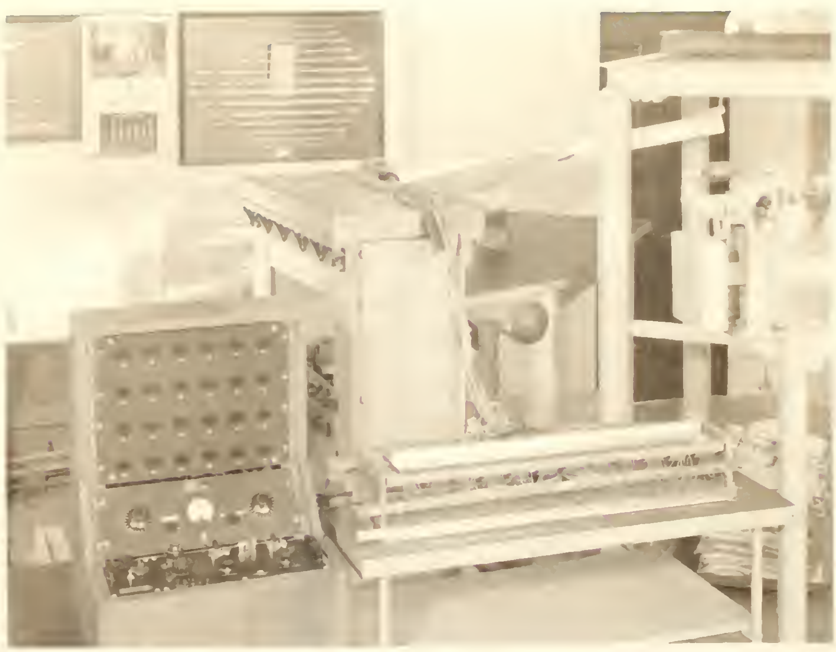
FIGURE 1.

EQUIPMENT UNDER CONCENTRATED LOAD

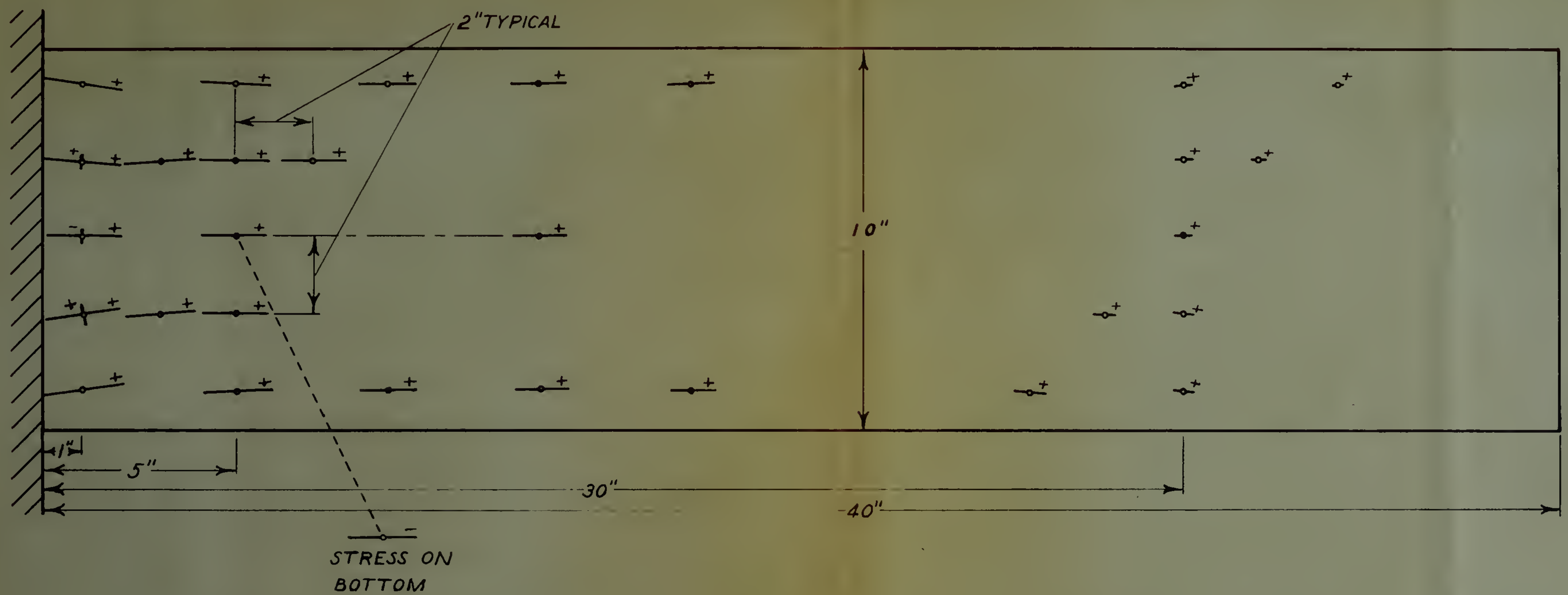
FIGURE 2.

EQUIPMENT UNDER TORSION LOAD

FIGURE 3.



ELECTRICAL EQUIPMENT



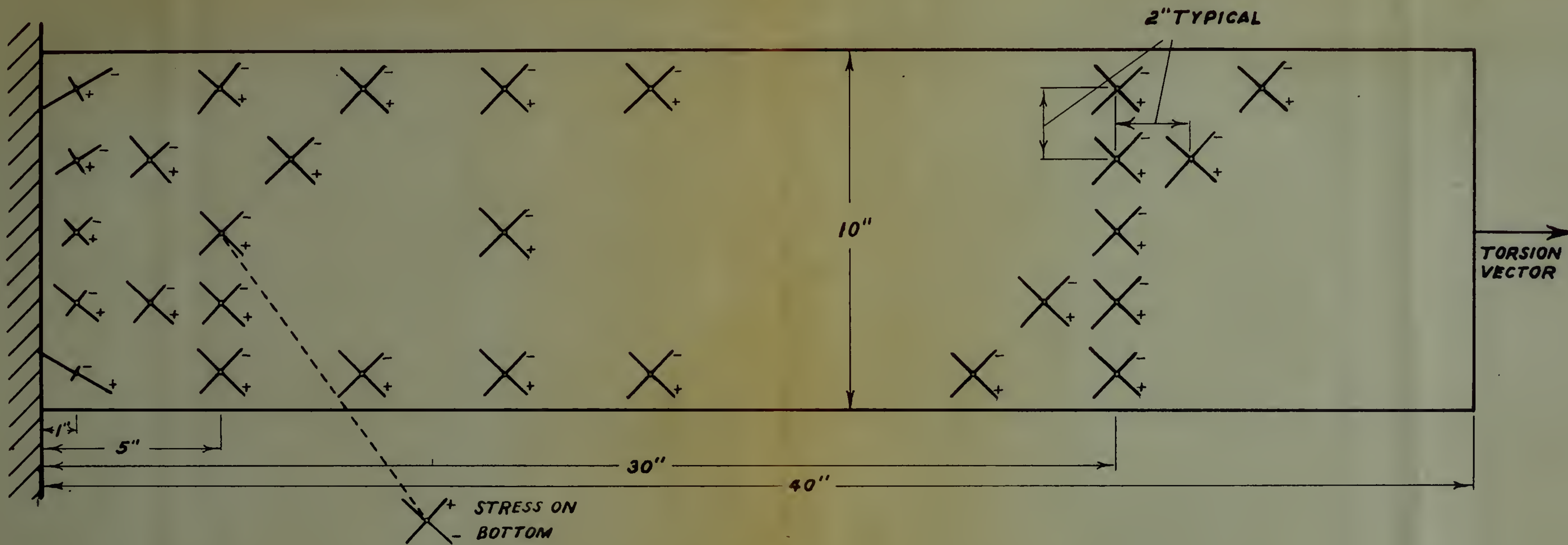
STRESSES PRODUCED BY 600# LOAD
UNIFORMLY DISTRIBUTED AT TIP
$\beta = 0°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20×10^3 P.S.I.

MODEL #1 TEST #1

Figure 4

RSC	FBG		
FBG	RSC		2-21-49
PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE			



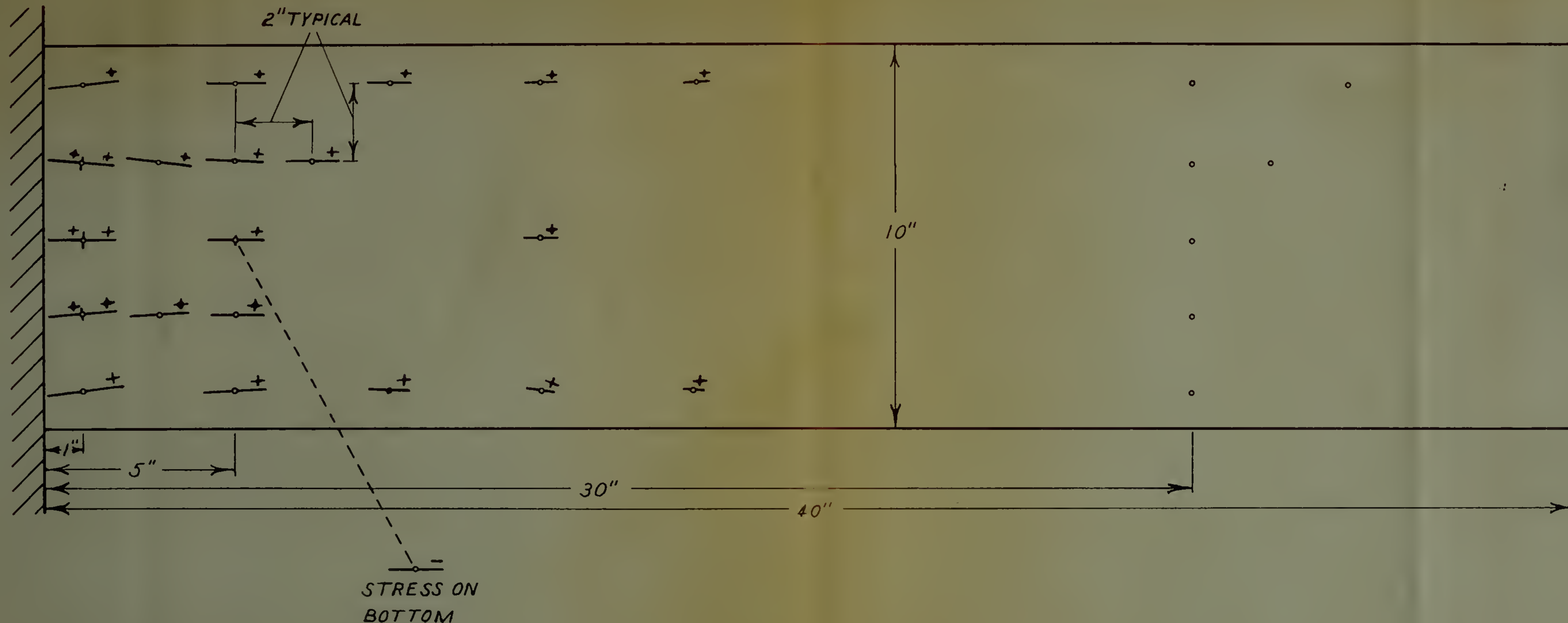
STRESSES PRODUCED BY 45000 IN.-LBS.
TORSION VECTOR PERPENDICULAR TO TIP
$\beta = 0^\circ$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. SCALE: 1" = 20X10³ P.S.I.

MODEL #1 TEST #2

Figure 5

FBG	RSC	
RSC	FBG	2-21-49
GUGGENHEIM AERONAUTICAL LABORATORY CALIFORNIA INSTITUTE OF TECHNOLOGY	PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE	



STRESSES PRODUCED BY LOAD OF 3 P.S.I. UNIFORMLY DISTRIBUTED OVER PLATE

$\beta = 0°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGN INDICATES TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20×10^3 P.S.I.

MODEL #1 TEST # 3

Figure 6

F B G	R S C
R S C	F B G

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

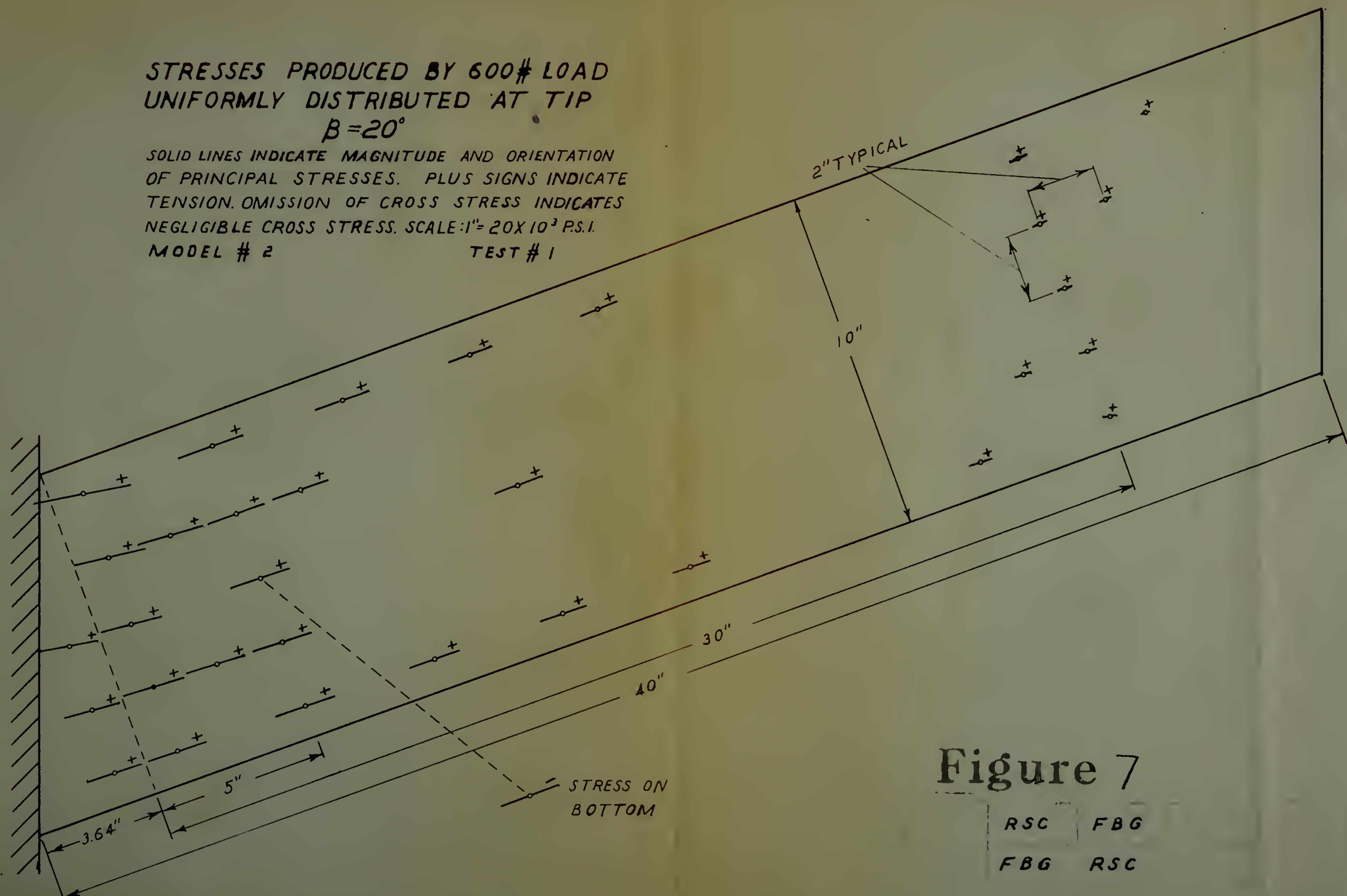


Figure 7

RSC FBG
FBG RSC

2-21-49

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

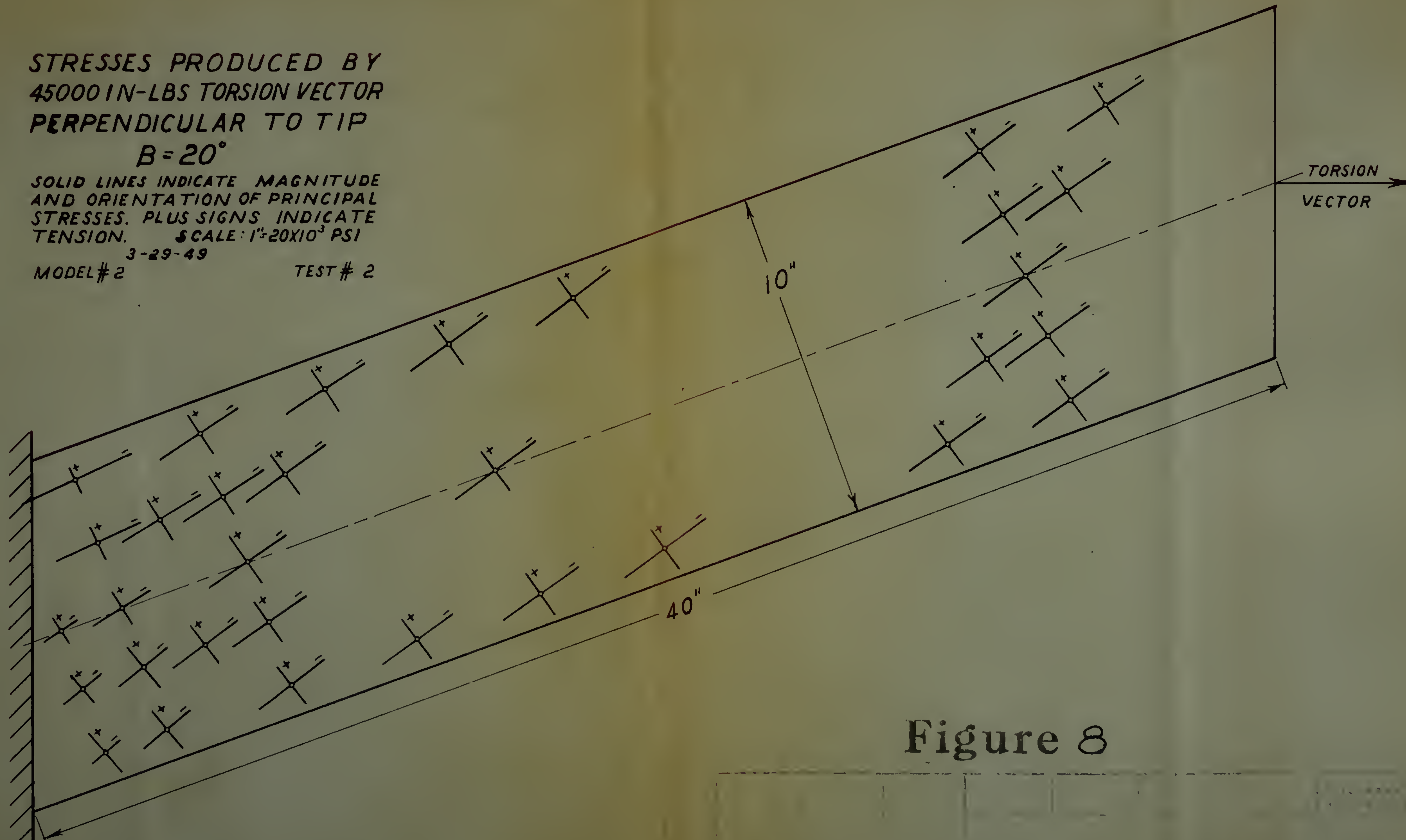


Figure 8

PRINCIPAL STRESSES IN
CANTILEVER SWEPT PLATE

STRESSES PRODUCED BY LOAD OF 3 P.S.I.
UNIFORMLY DISTRIBUTED OVER PLATE
$\beta = 20°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20X10³ P.S.I.

MODEL #2 TEST #3

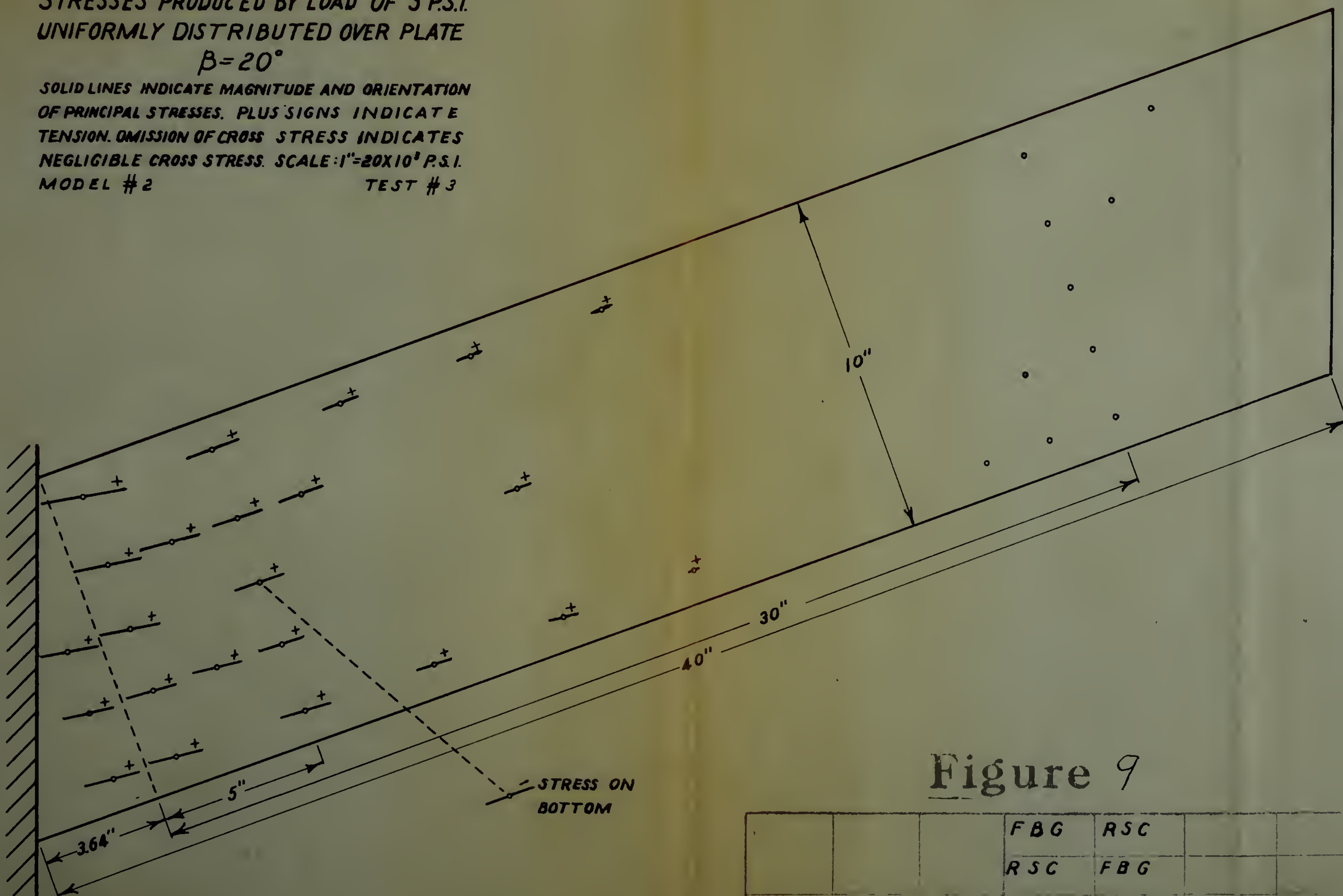


Figure 9

			FBG	RSC			
			RSC	FBG			
MATERIAL							

GUGGENHEIM AERONAUTICAL LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

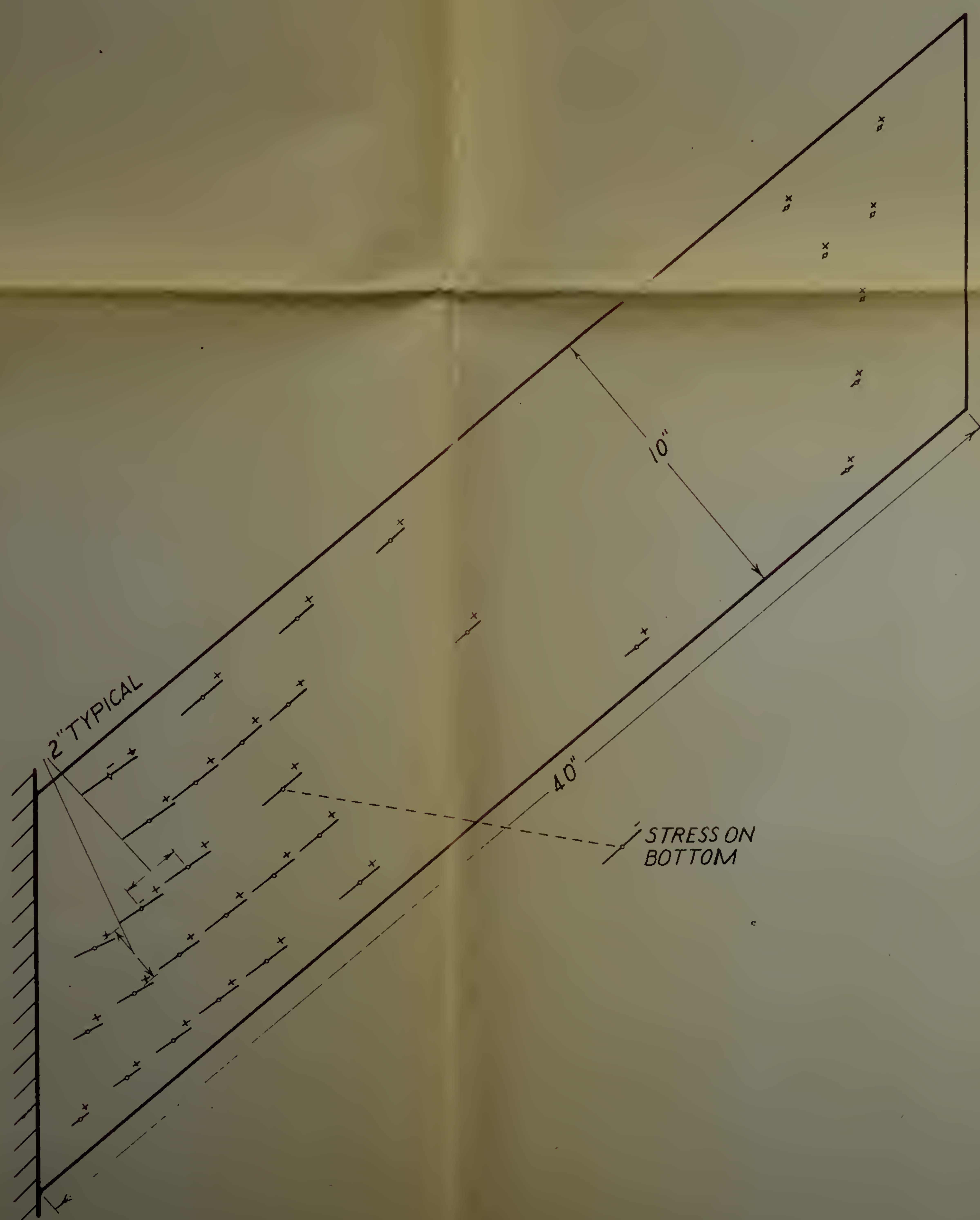


Figure 10

STRESSES PRODUCED BY 600# LOAD
UNIFORMLY DISTRIBUTED AT TIP
$\beta = 40^\circ$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: $1'' = 20 \times 10^3$ PSI

MODEL # 3 TEST # 1

PRINCIPAL STRESSES IN
CANTILEVER SWEPT PLATE

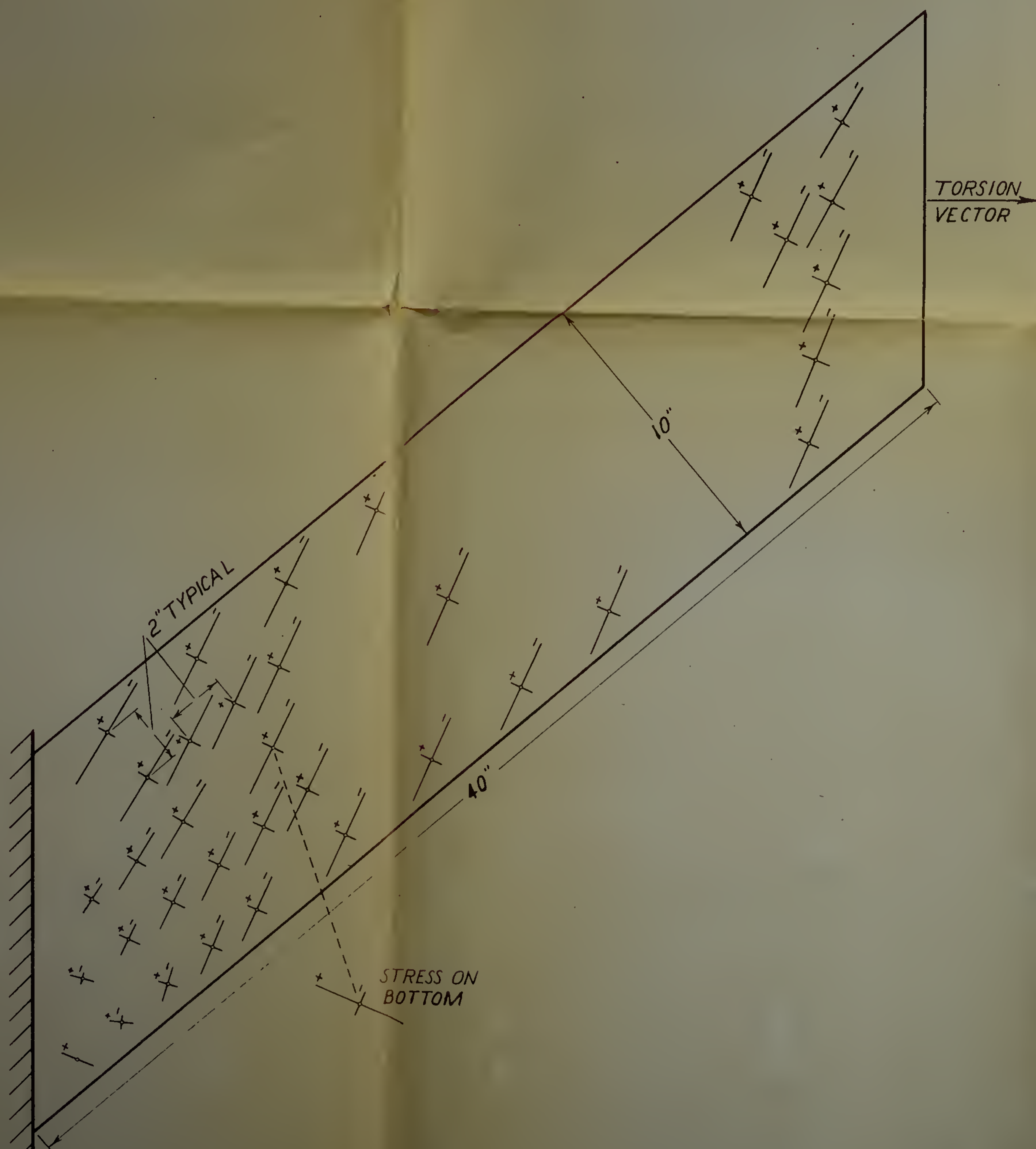


Figure 11

STRESSES PRODUCED BY
45000 IN-LBS TORSION VECTOR
PERPENDICULAR TO TIP
$\beta = 40°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION.
SCALE: $1'' = 20 \times 10^3$ PSI

MODEL #3 TEST #2

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

Figure 12

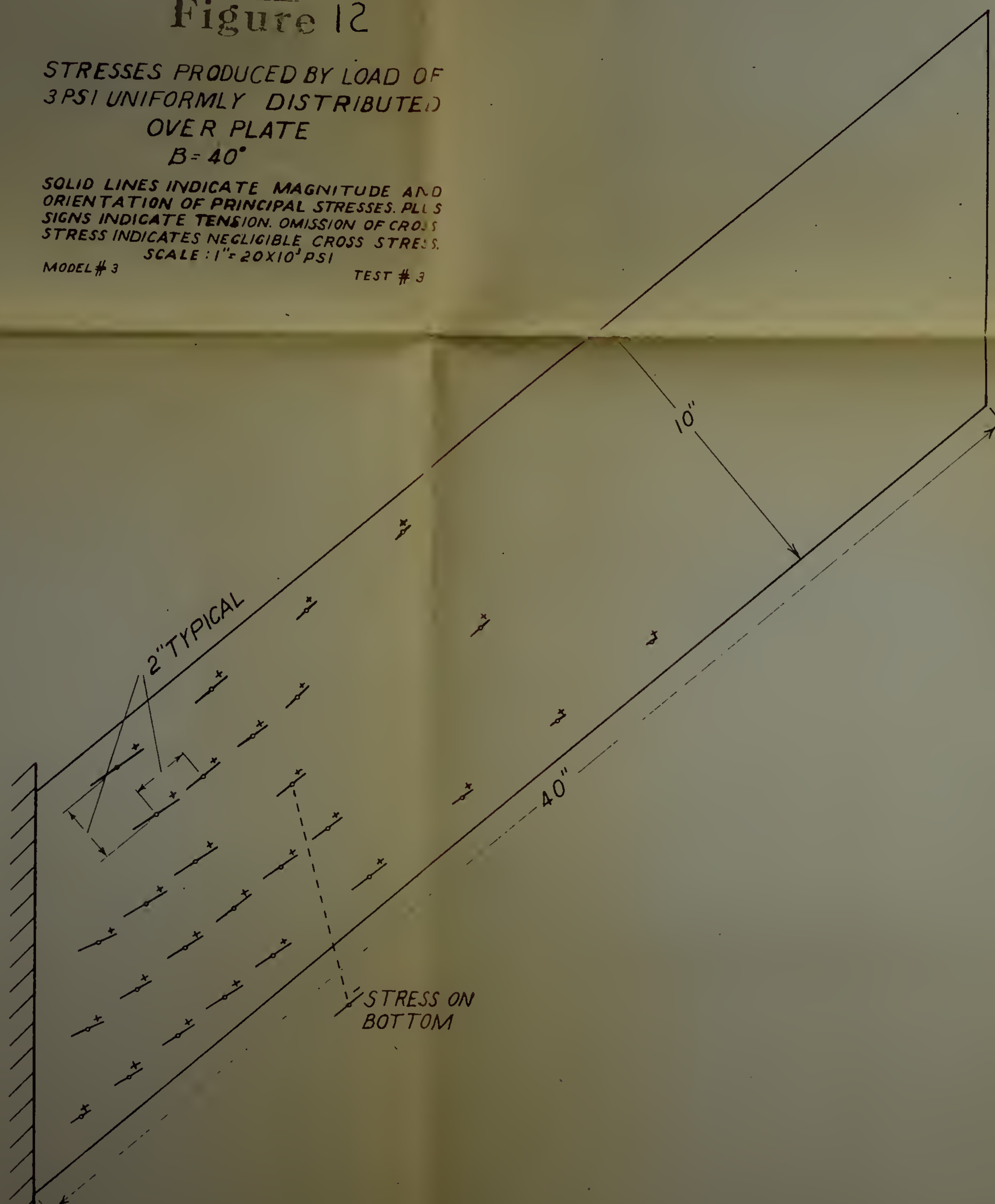
STRESSES PRODUCED BY LOAD OF 3 PSI UNIFORMLY DISTRIBUTED OVER PLATE

$\beta = 40°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS.

SCALE : 1" = 20×10^3 PSI

MODEL # 3 TEST # 3



PRINCIPAL STRESSES IN
CANTILEVER SWEPT PLATE

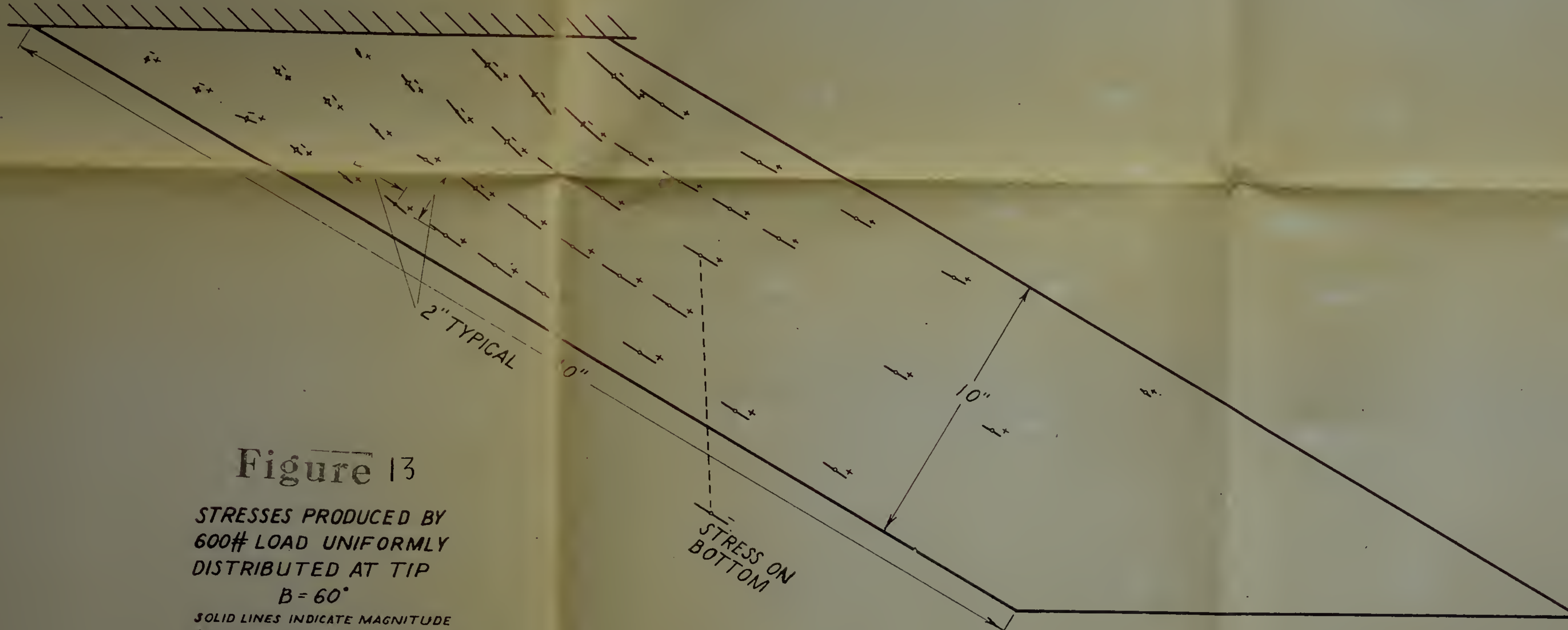


Figure 13

STRESSES PRODUCED BY 600# LOAD UNIFORMLY DISTRIBUTED AT TIP

B = 60°

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPLE STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS. SCALE: 1" = 20X10⁴PSI

MODEL # 4 TEST # 1

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

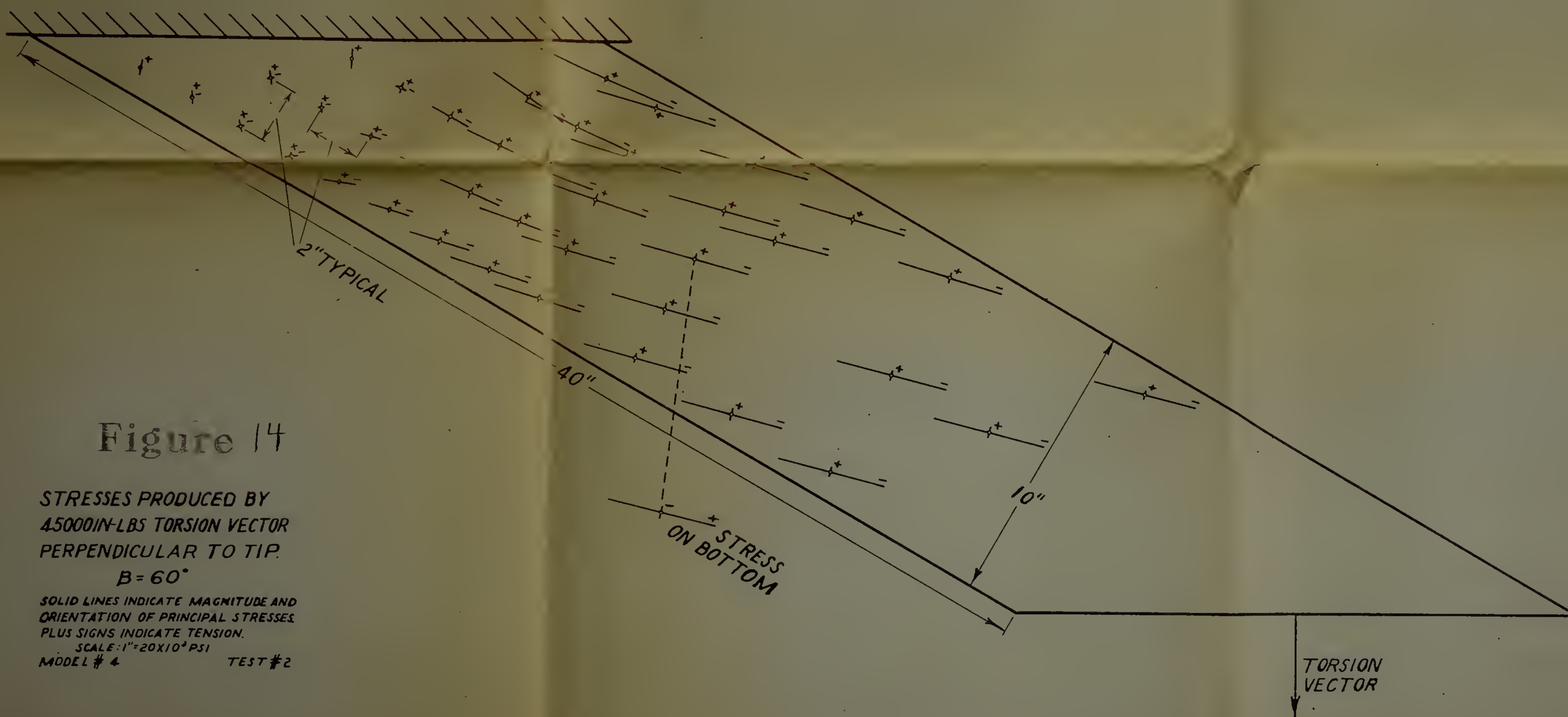


Figure 14

STRESSES PRODUCED BY 45000 IN-LBS TORSION VECTOR PERPENDICULAR TO TIP.
$B = 60°$

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES.
PLUS SIGNS INDICATE TENSION.
SCALE: $1" = 20 \times 10^3$ PSI
MODEL # 4 TEST # 2

PRINCIPAL STRESSES IN CANTILEVER SWEPT PLATE

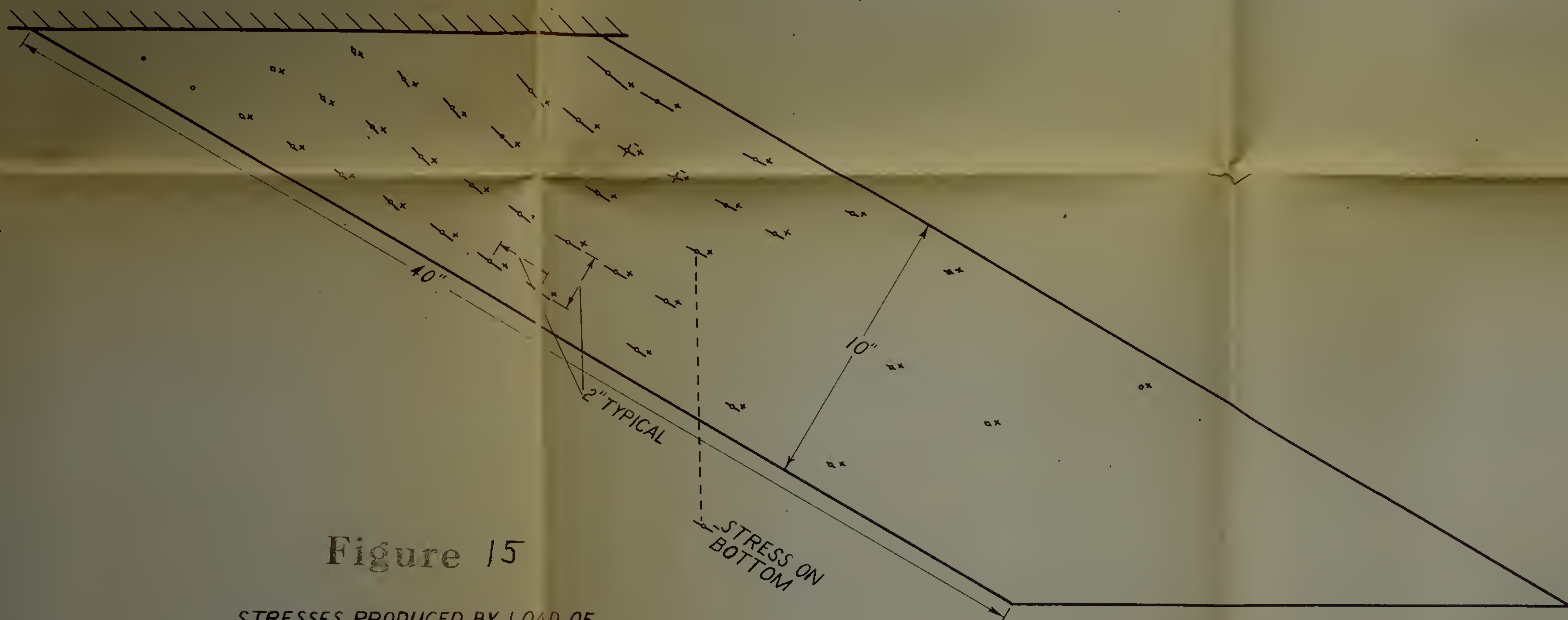


Figure 15

STRESSES PRODUCED BY LOAD OF
3PSI UNIFORMLY DISTRIBUTED
OVER PLATE
B = 60°

SOLID LINES INDICATE MAGNITUDE AND ORIENTATION OF PRINCIPAL STRESSES. PLUS SIGNS INDICATE TENSION. OMISSION OF CROSS STRESS INDICATES NEGLIGIBLE CROSS STRESS.
SCALE: 1" = 20X10³ PSI

MODEL # 4 TEST # 3

PRINCIPAL STRESSES IN
CANTILEVER SWEPT PLATE

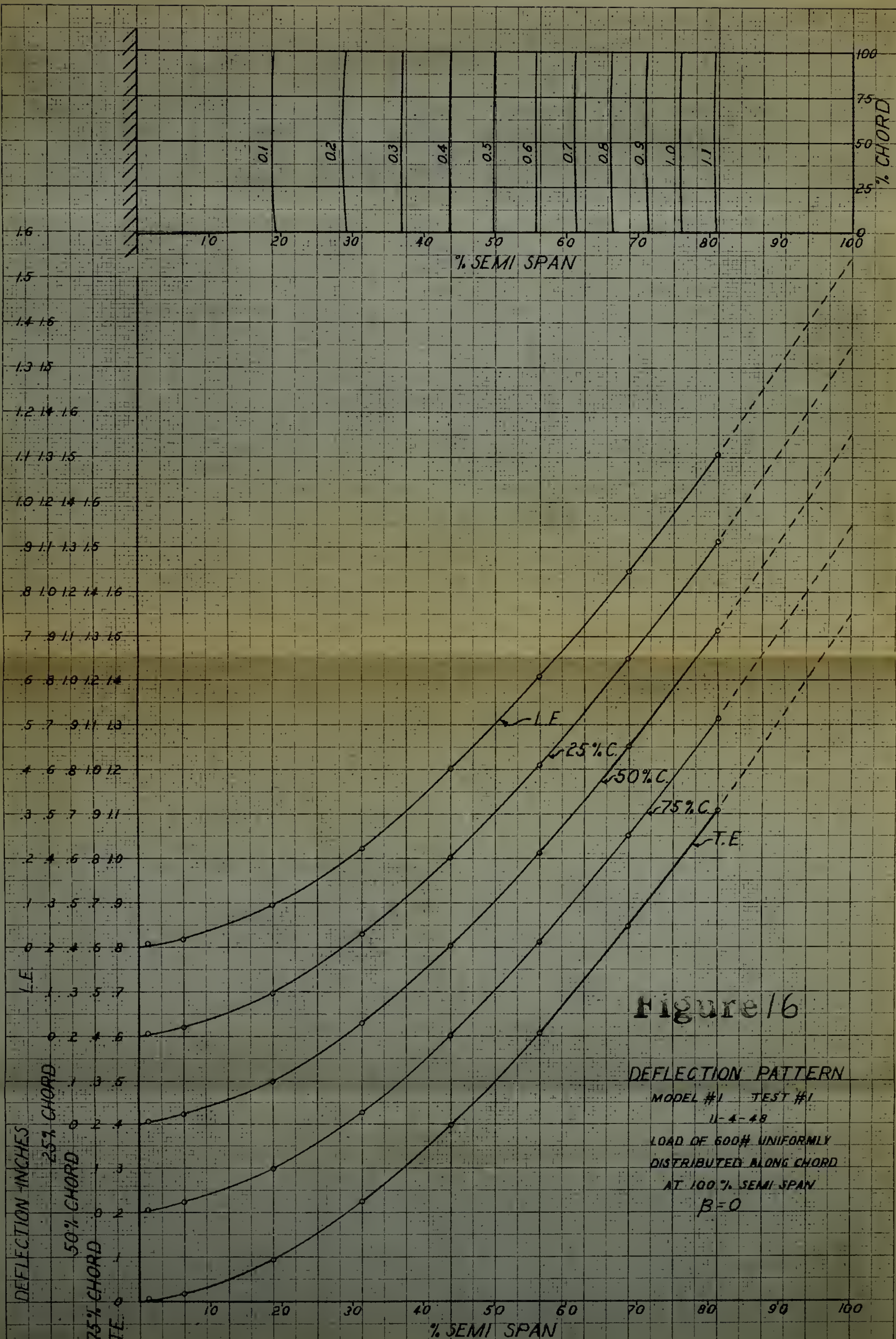


Figure 16

DEFLECTION PATTERN
MODEL #1 TEST #1
11-4-48
LOAD OF 600# UNIFORMLY
DISTRIBUTED ALONG CHORD
AT 100% SEMI SPAN
$\beta = 0$

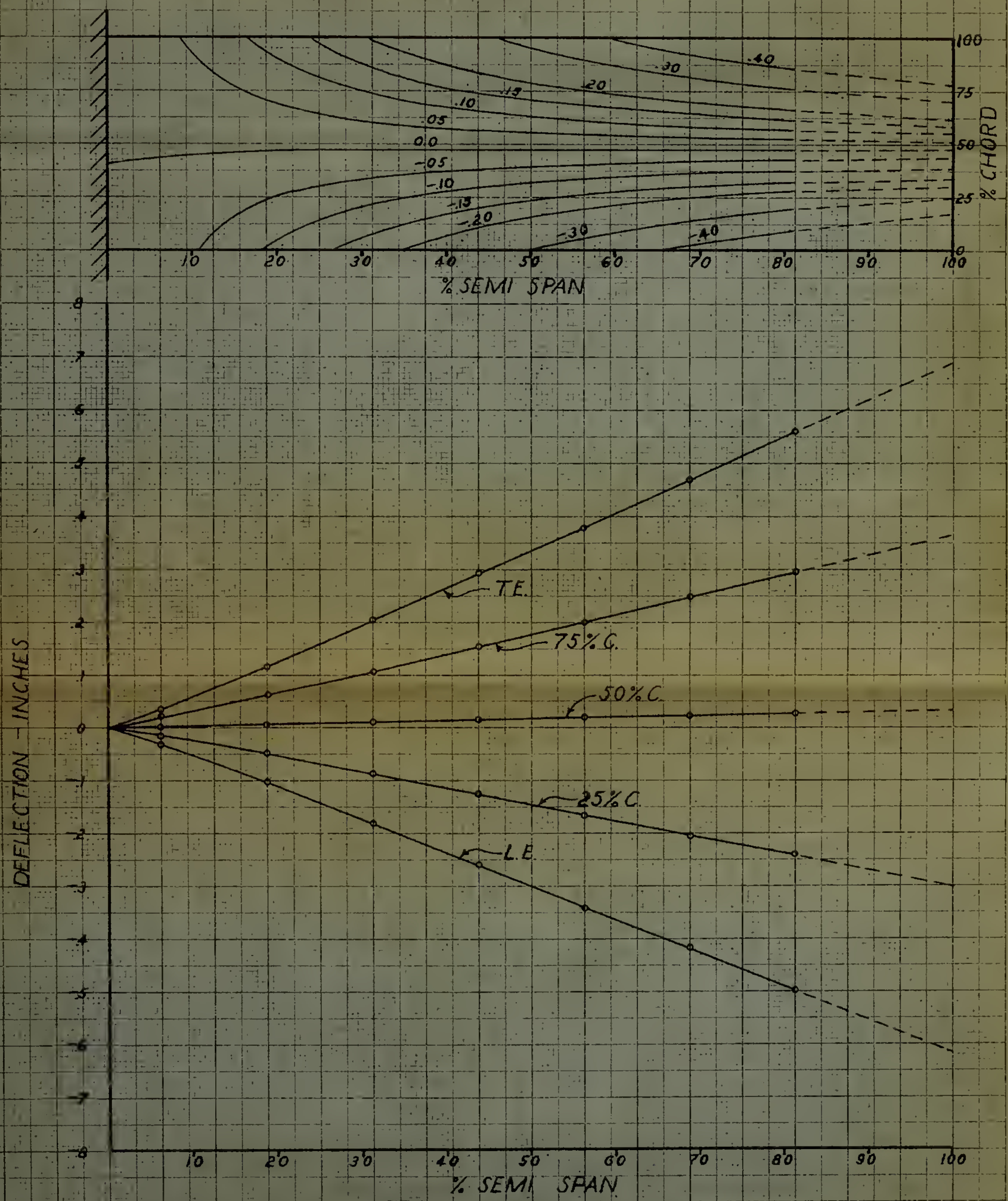


Figure 17

DEFLECTION PATTERN

MODEL #1 TEST #2

12-5-48

LOADED AT TIP WITH TORSIONAL MOMENT

VECTOR OF 45000 IN-# PERPENDICULAR TO ROOT

B=0°

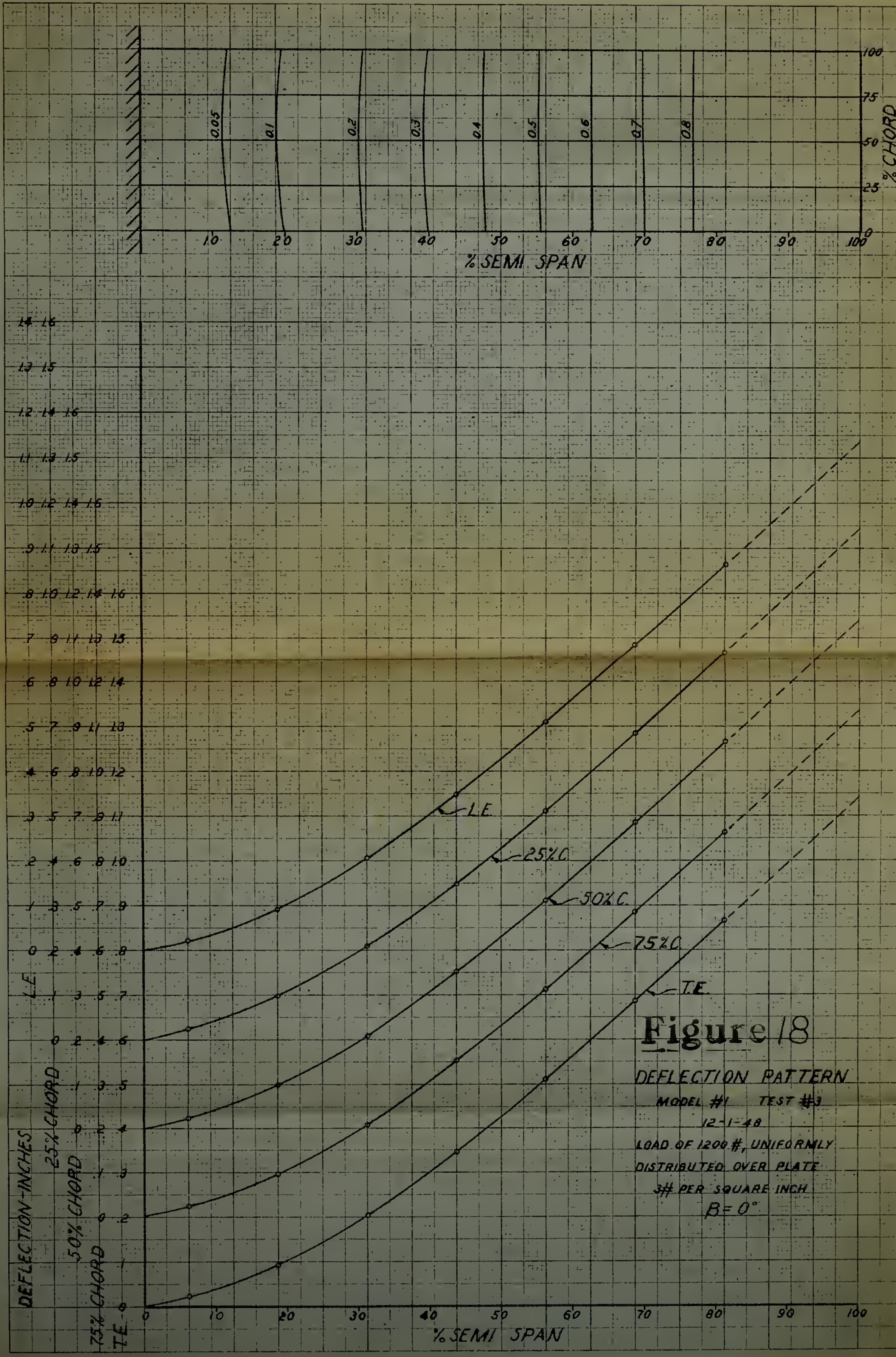


Figure 18

DEFLECTION PATTERN

MODEL #1 TEST #3

12-1-48

LOAD OF 1200 #, UNIFORMLY DISTRIBUTED OVER PLATE

3# PER SQUARE INCH

$\beta = 0°$

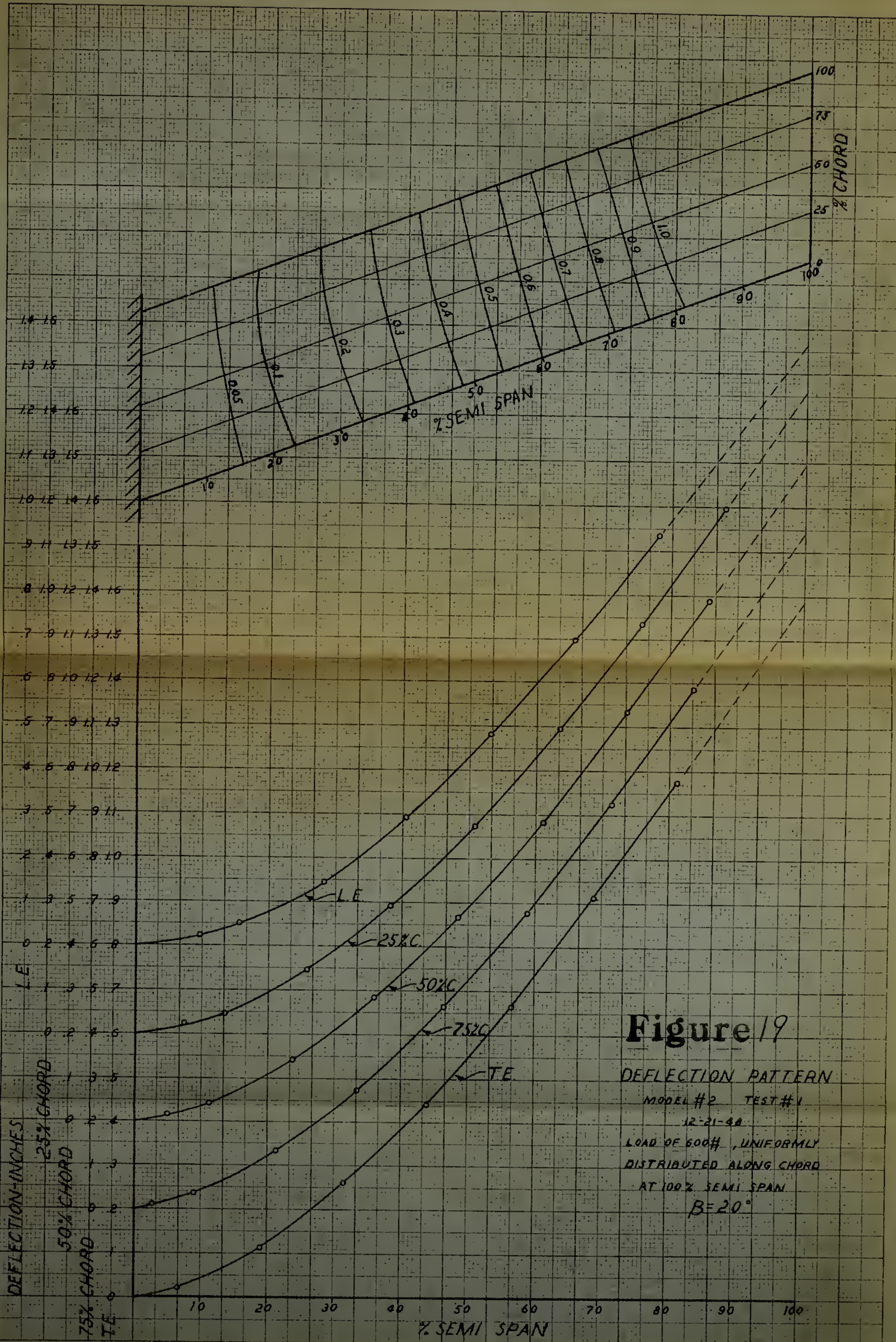


Figure 19

DEFLECTION PATTERN

MODEL #2 TEST #1

12-21-48

LOAD OF 600# , UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN

$\beta = 20°$

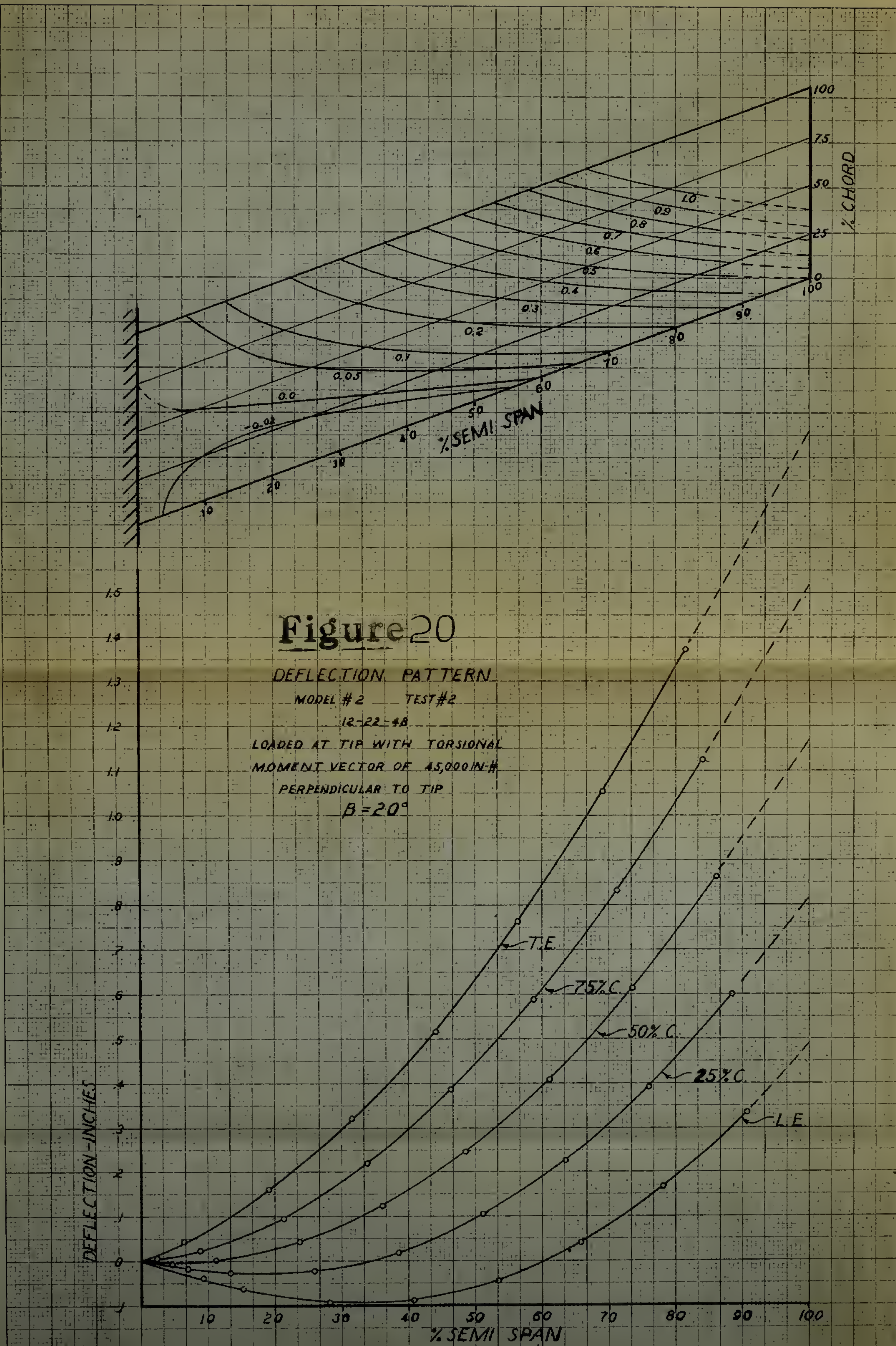


Figure 20

DEFLECTION PATTERN

MODEL #2 TEST #2

12-22-48

LOADED AT TIP WITH TORSIONAL MOMENT VECTOR OF 45,000 IN-# PERPENDICULAR TO TIP

$\beta = 20°$

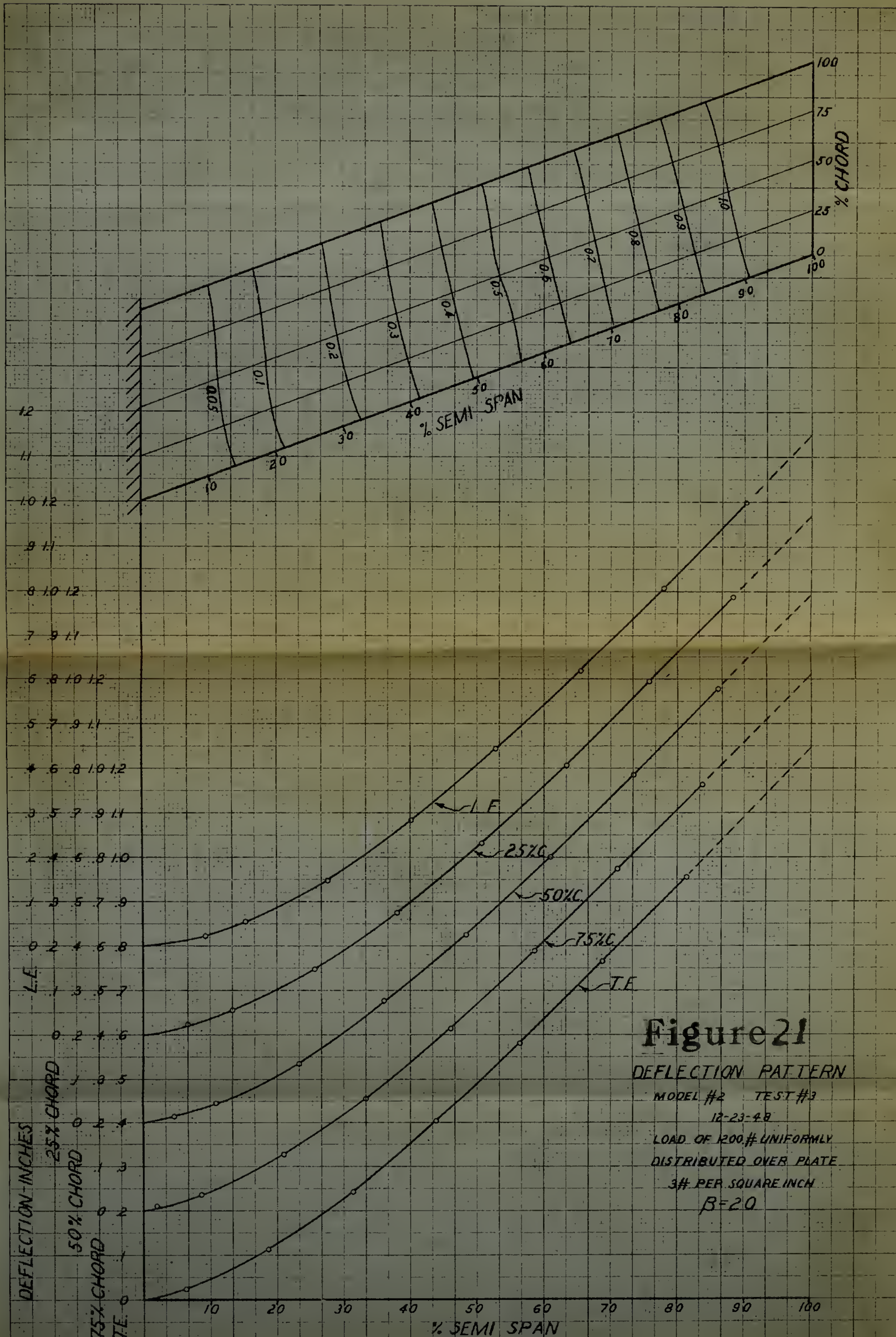


Figure 21

DEFLECTION PATTERN

MODEL #2 TEST #3

12-23-48

LOAD OF 1200 # UNIFORMLY DISTRIBUTED OVER PLATE

3# PER SQUARE INCH

$\beta = 20$

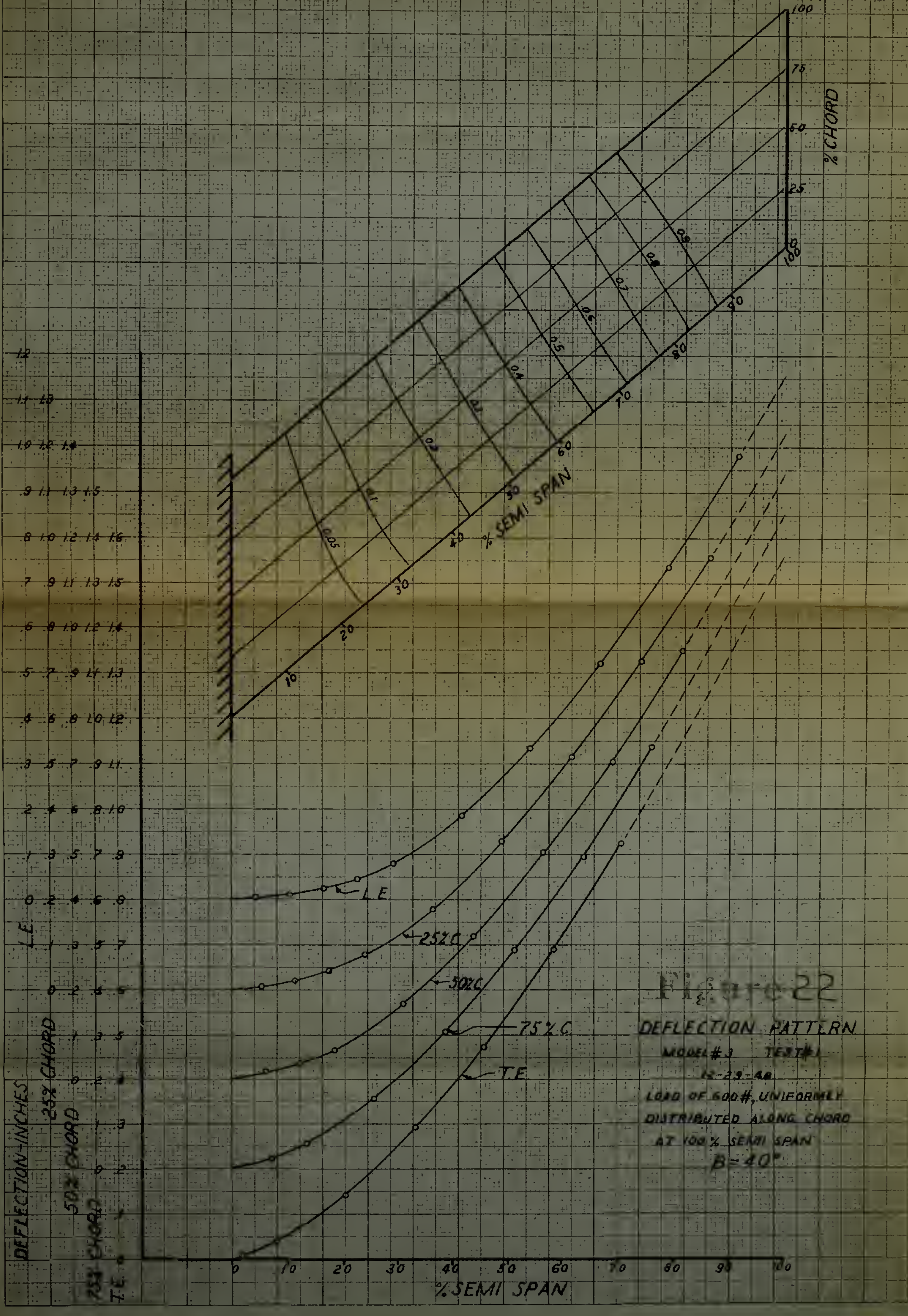


Figure 22

DEFLECTION PATTERN

MODEL #3 TEST #1

12-29-48

LOAD OF 600 #, UNIFORMLY DISTRIBUTED ALONG CHORD AT 100 % SEMI SPAN

$B = 40°$

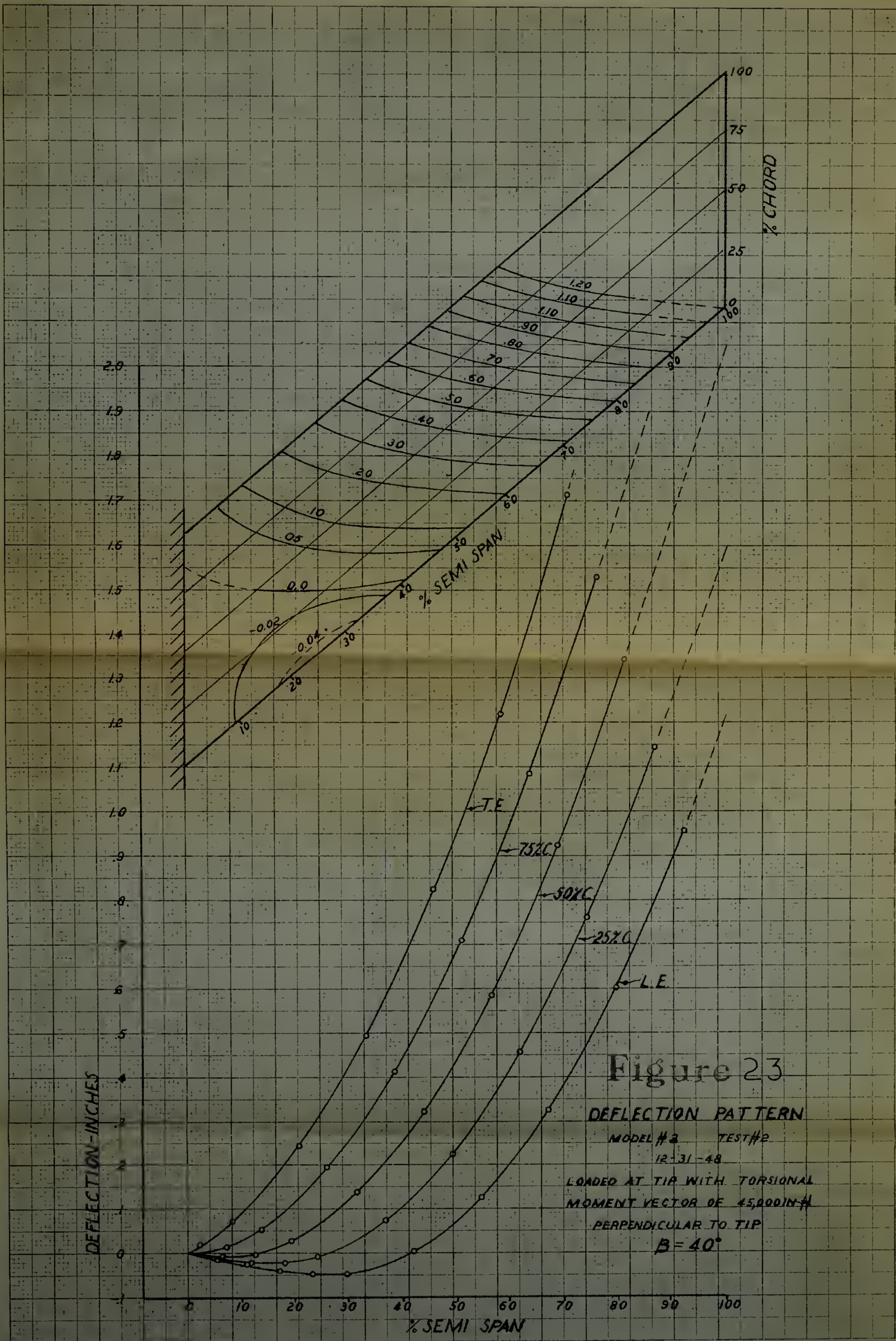


Figure 23

DEFLECTION PATTERN

MODEL #2 TEST #2

12-31-48

LOADED AT TIP WITH TORSIONAL MOMENT VECTOR OF 45,000 IN # PERPENDICULAR TO TIP

$B = 40°$

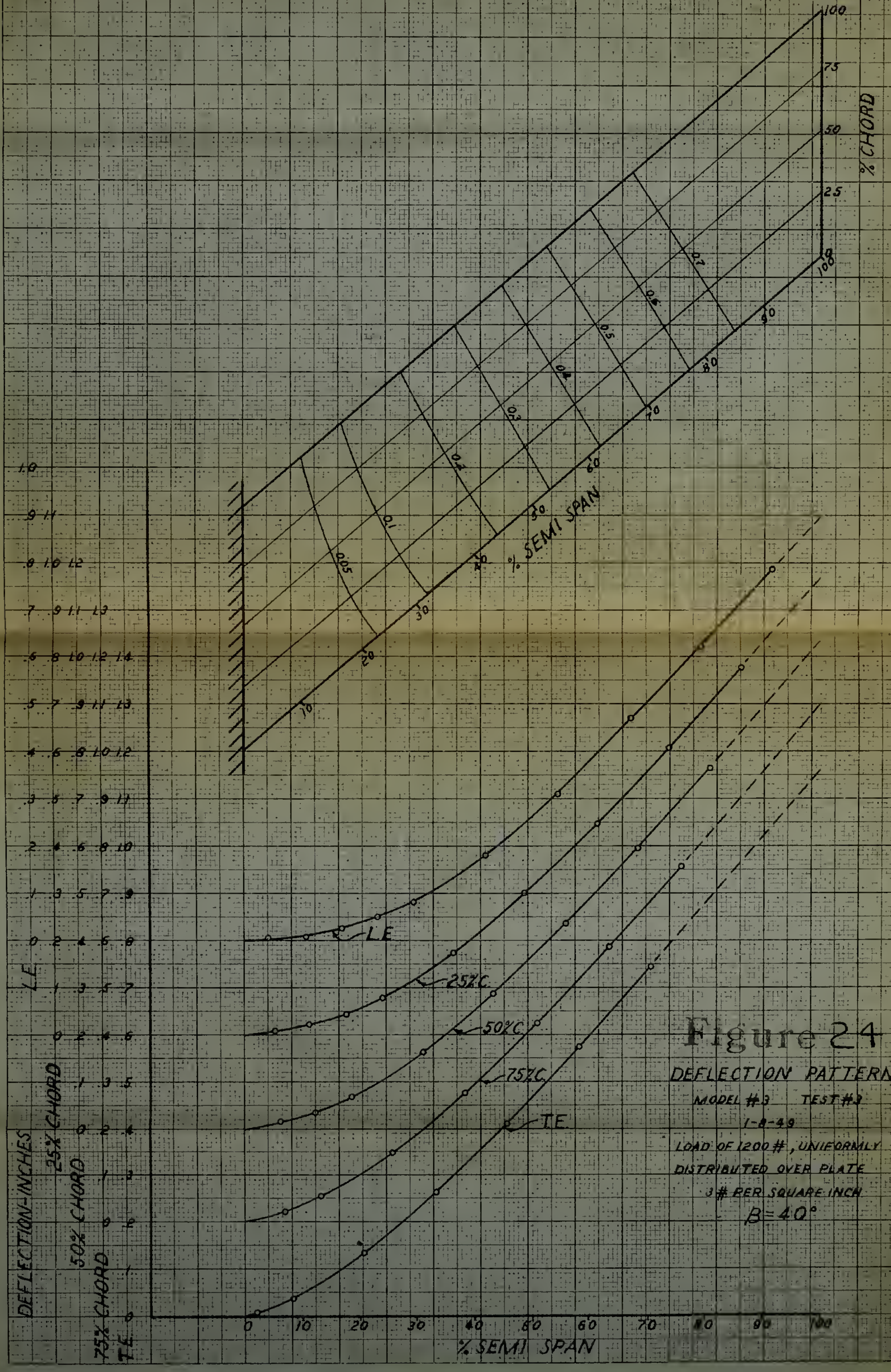


Figure 24

DEFLECTION PATTERN

MODEL #3 TEST #3

1-8-49

LOAD OF 1200 #, UNIFORMLY DISTRIBUTED OVER PLATE

3 # PER SQUARE INCH

$B = 40°$

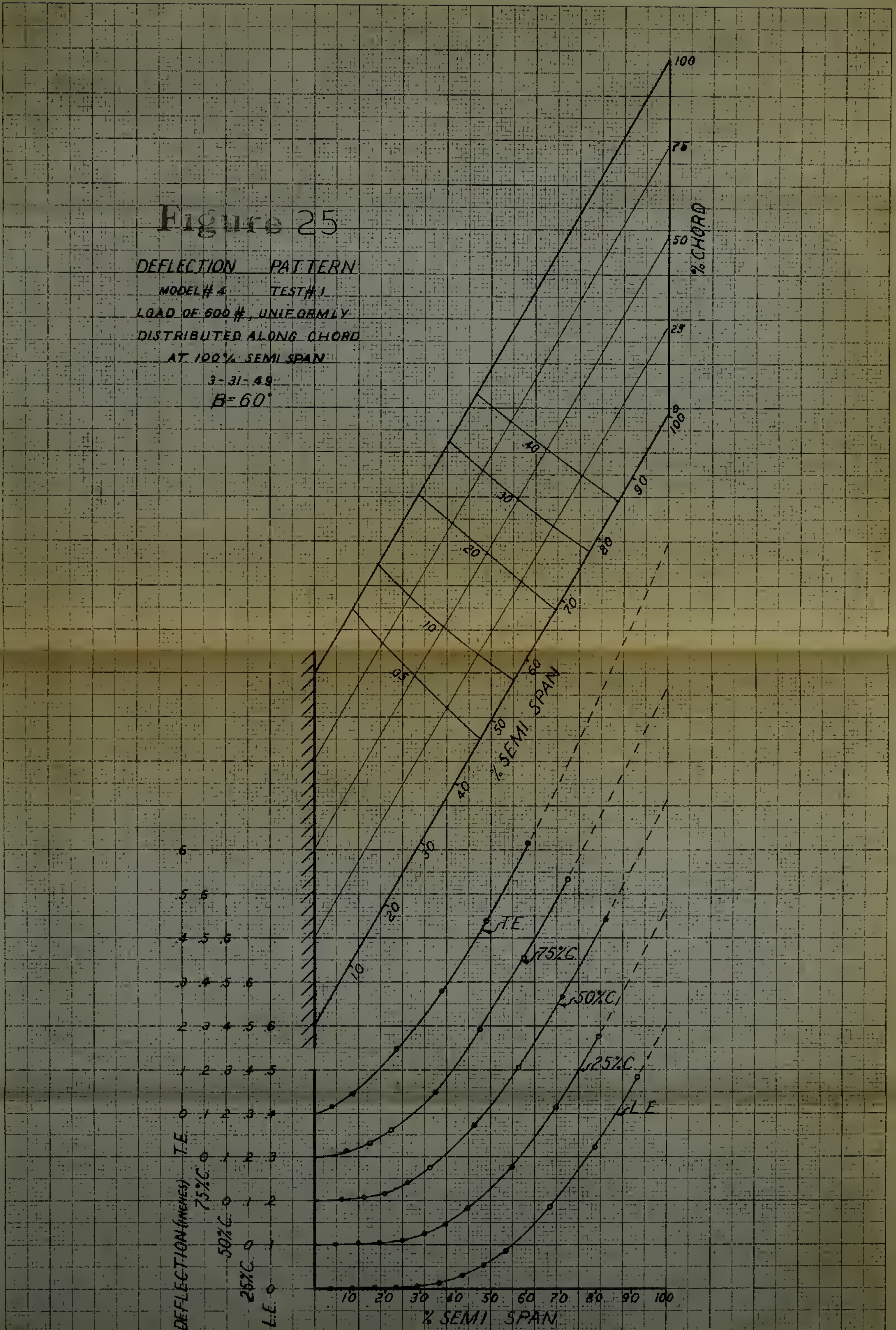


Figure 25
DEFLECTION PATTERN
MODEL # 4 TEST # 1
LOAD OF 600 #, UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN
3-31-49
β=60°
% CHORD
% SEMI SPAN
T.E.
75%C.
50%C.
25%C.
L.E.
DEFLECTION (INCHES)
% SEMI SPAN

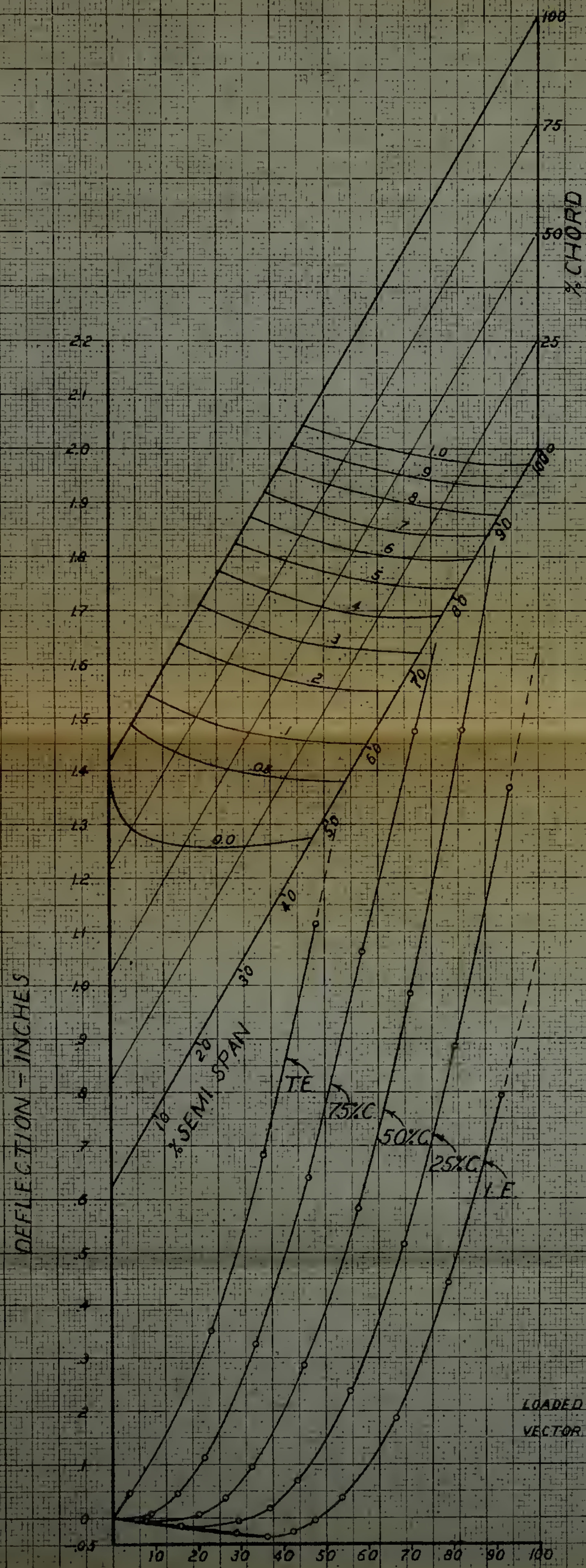


Figure 26

DEFLECTION PATTERN

MODEL # 4 TEST # 2

LOADED AT TIP WITH TORSIONAL MOMENT VECTOR OF 45,000 IN # PERPENDICULAR TO ROOT

$\beta = 60°$

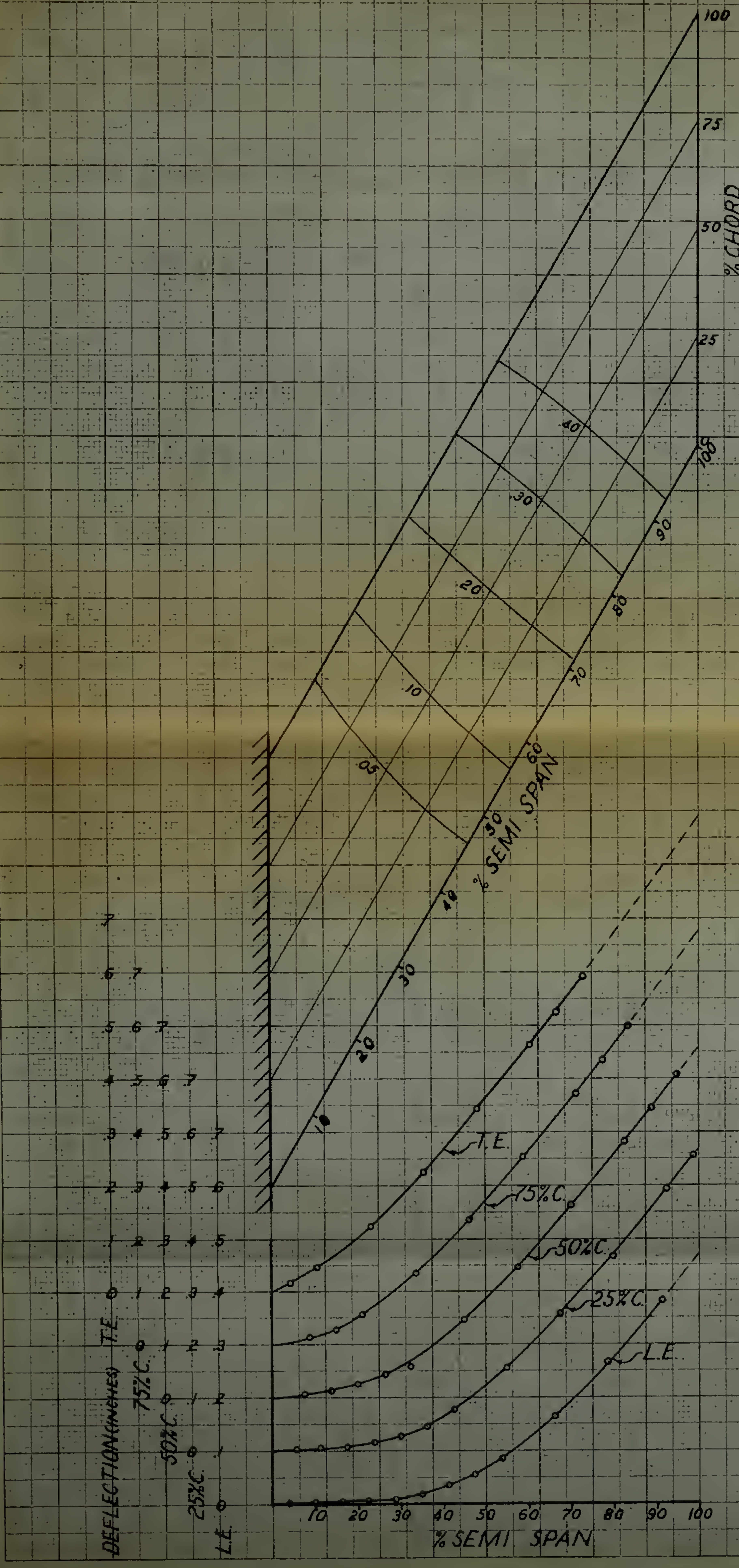


Figure 27

DEFLECTION PATTERN

MODEL #4 TEST #3

3-30-49

LOAD OF 1200#, UNIFORMLY DISTRIBUTED OVER PLATE

3# PER SQUARE IN.

$\beta = 60°$

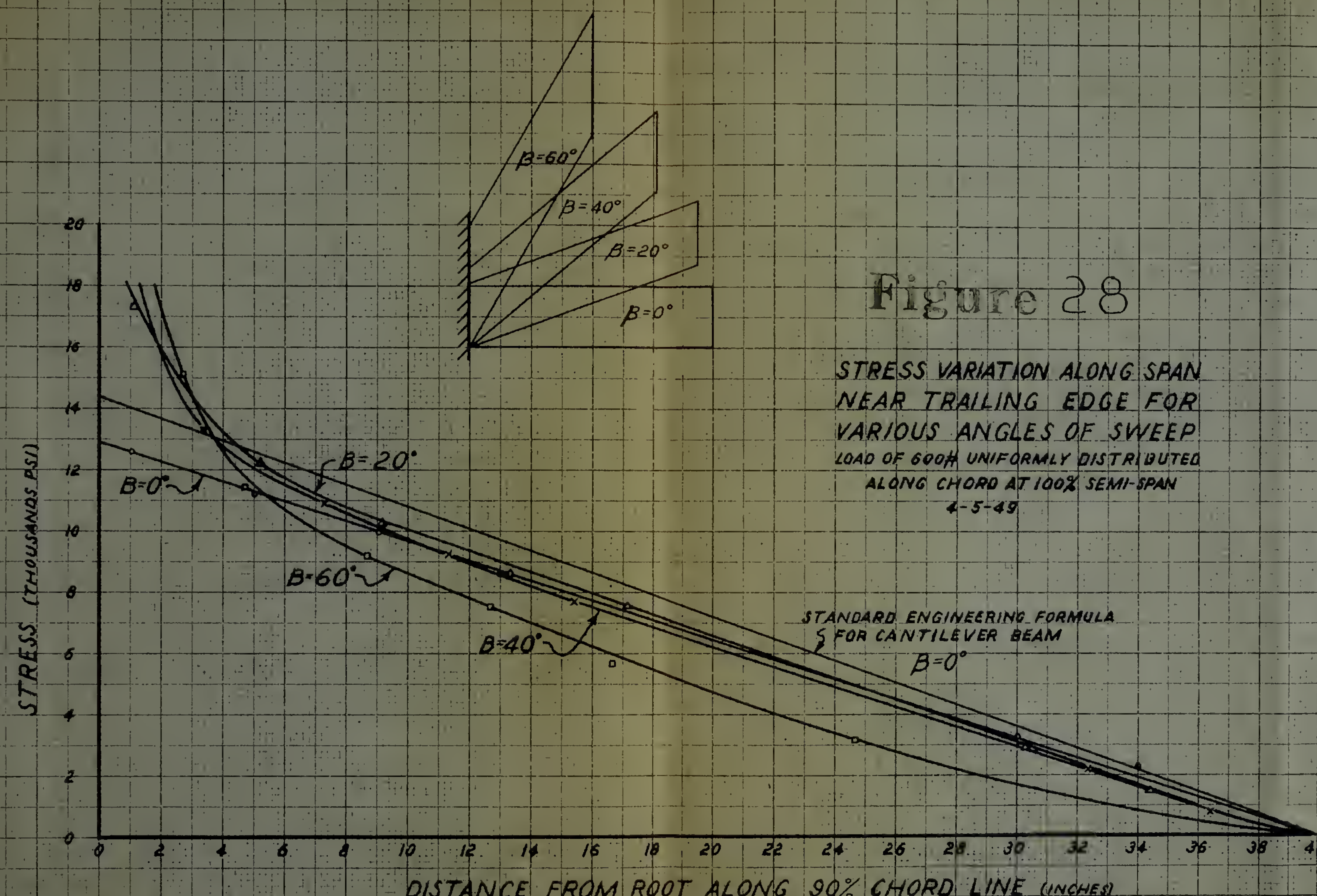


Figure 28
STRESS VARIATION ALONG SPAN
NEAR TRAILING EDGE FOR
VARIOUS ANGLES OF SWEEP
LOAD OF 600# UNIFORMLY DISTRIBUTED
ALONG CHORD AT 100% SEMI-SPAN
4-5-49
β=60°
β=40°
β=20°
β=0°
B=20°
B=0°
B=60°
B=40°
STANDARD ENGINEERING FORMULA
FOR CANTILEVER BEAM
β=0°
STRESS (THOUSANDS PSI)
DISTANCE FROM ROOT ALONG 90% CHORD LINE (INCHES)
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
36
38
40

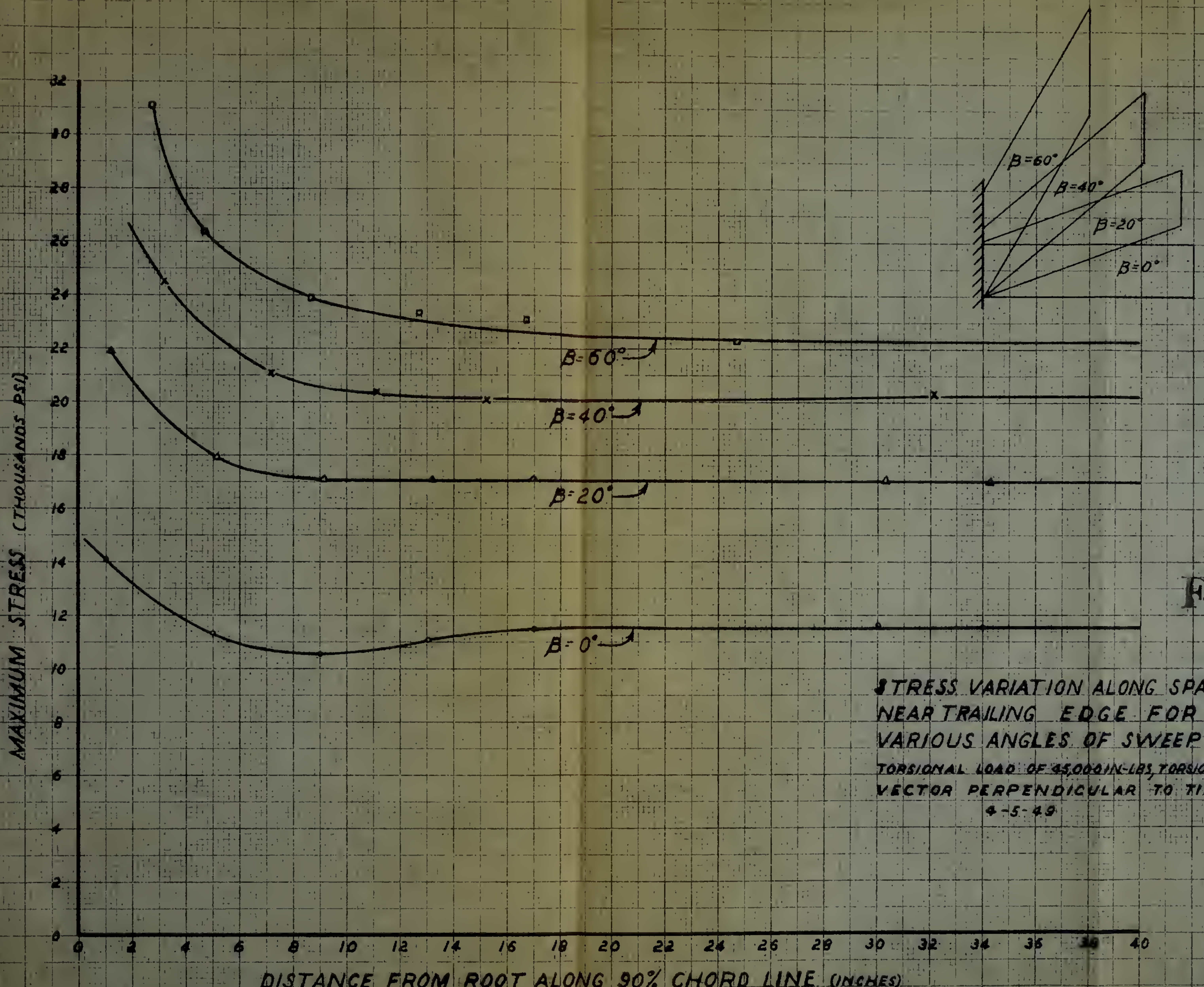


Figure 29

STRESS VARIATION ALONG SPAN NEAR TRAILING EDGE FOR VARIOUS ANGLES OF SWEEP

TORSIONAL LOAD OF 45,000 IN-LBS, TORSION VECTOR PERPENDICULAR TO TIP

4-5-49

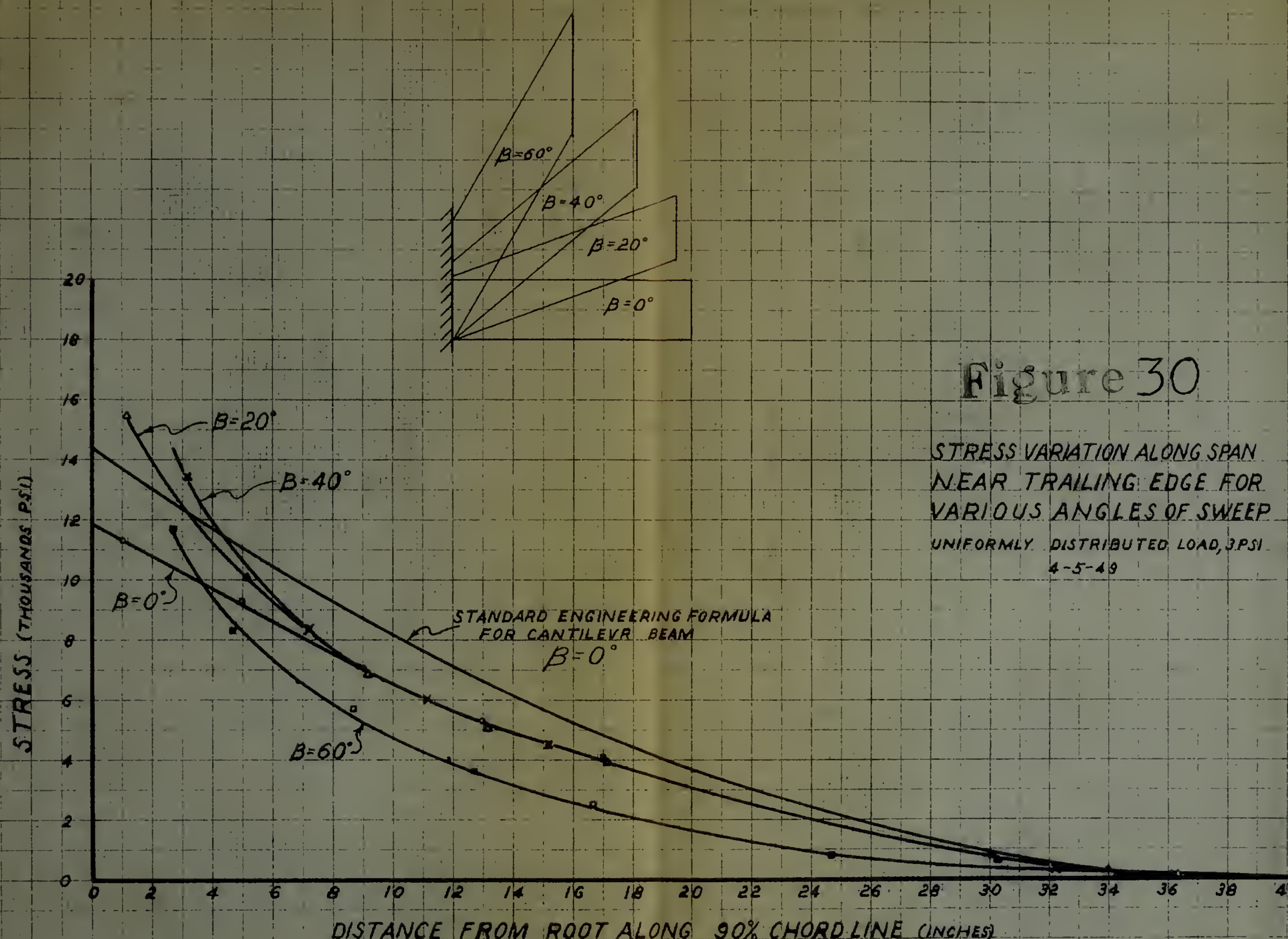


Figure 30

STRESS VARIATION ALONG SPAN
NEAR TRAILING EDGE FOR
VARIOUS ANGLES OF SWEEP

UNIFORMLY DISTRIBUTED LOAD, 3 PSI
4-5-49

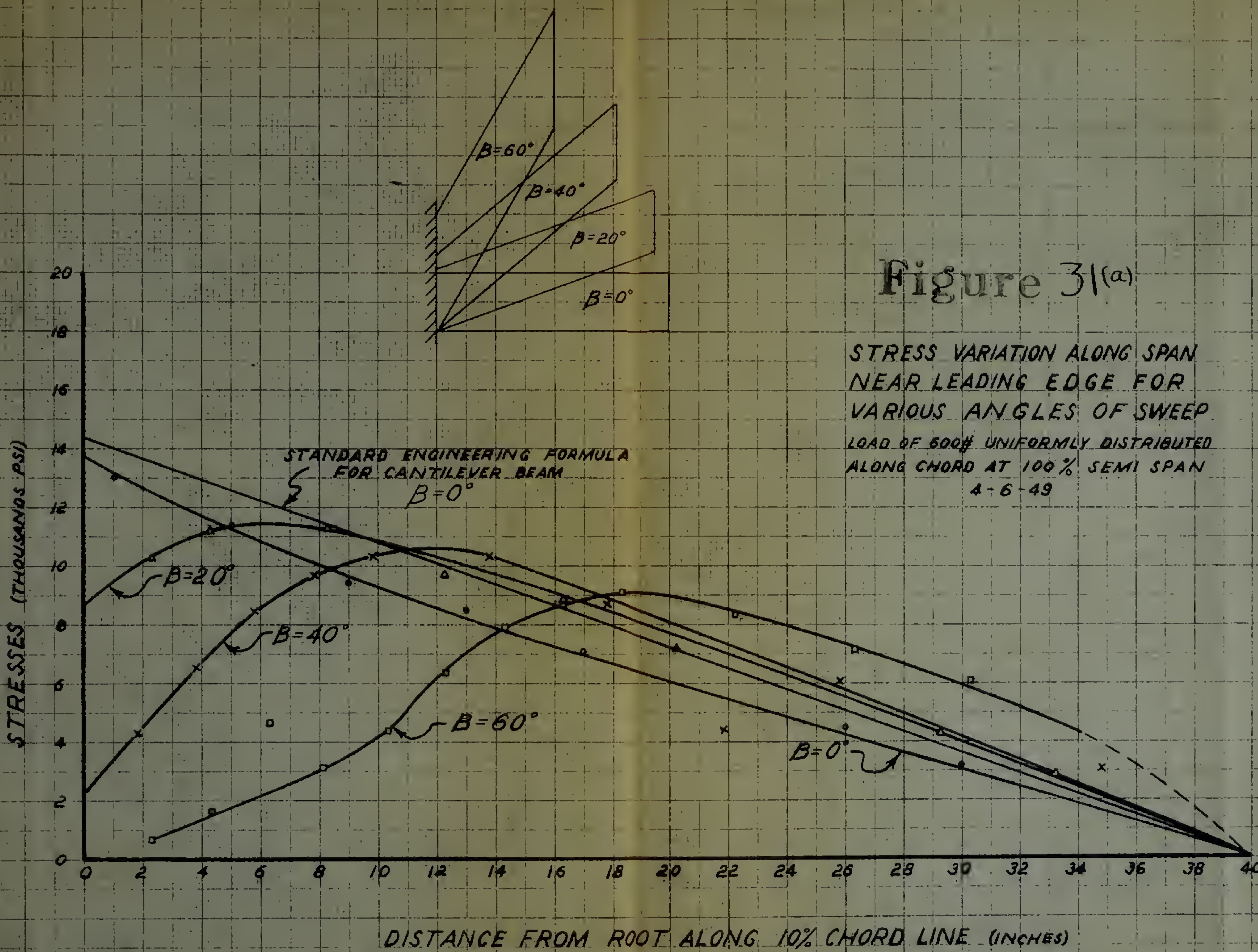


Figure 31(a)

STRESS VARIATION ALONG SPAN NEAR LEADING EDGE FOR VARIOUS ANGLES OF SWEEP

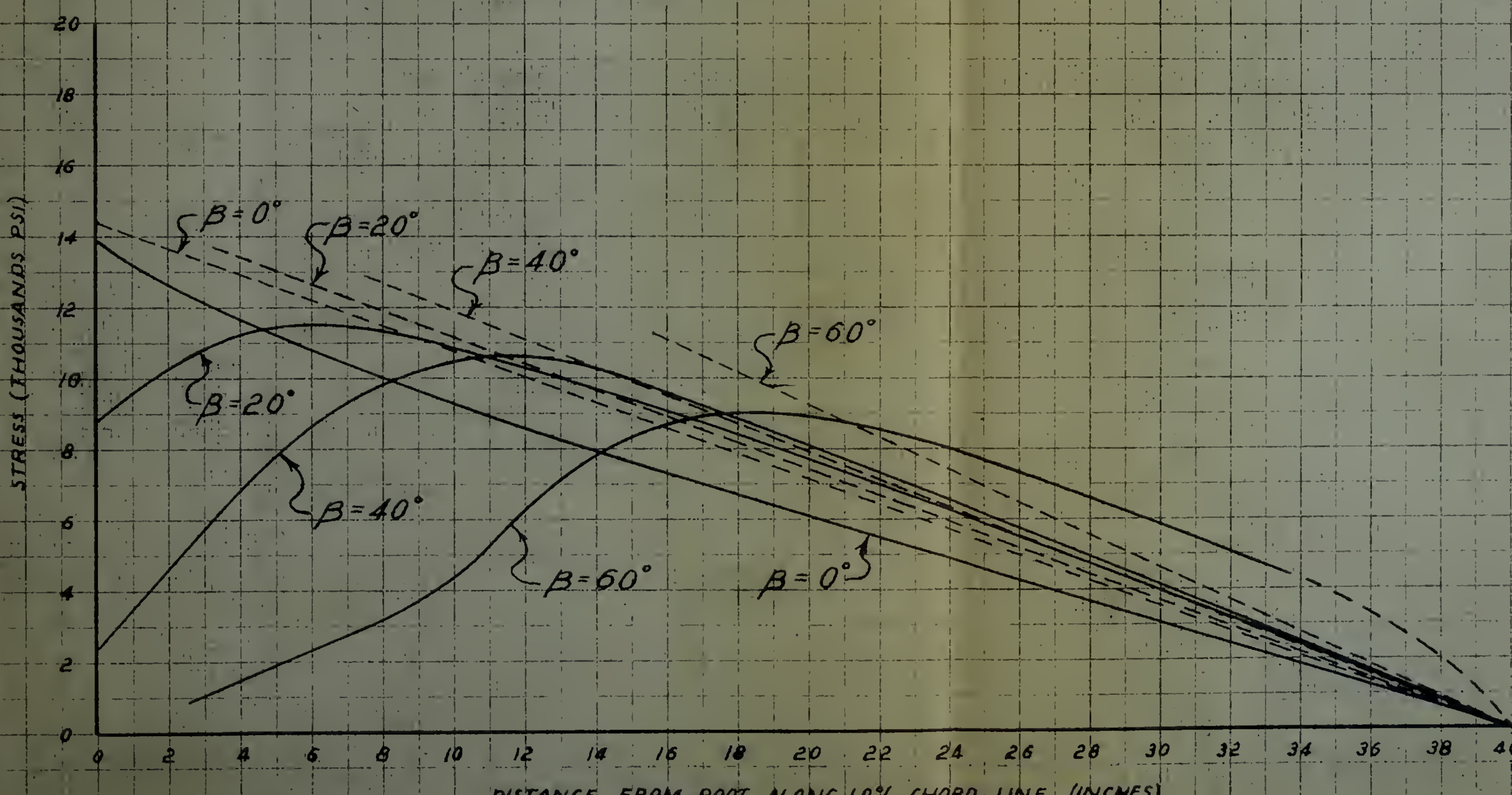
LOAD OF 600# UNIFORMLY DISTRIBUTED ALONG CHORD AT 100% SEMI SPAN

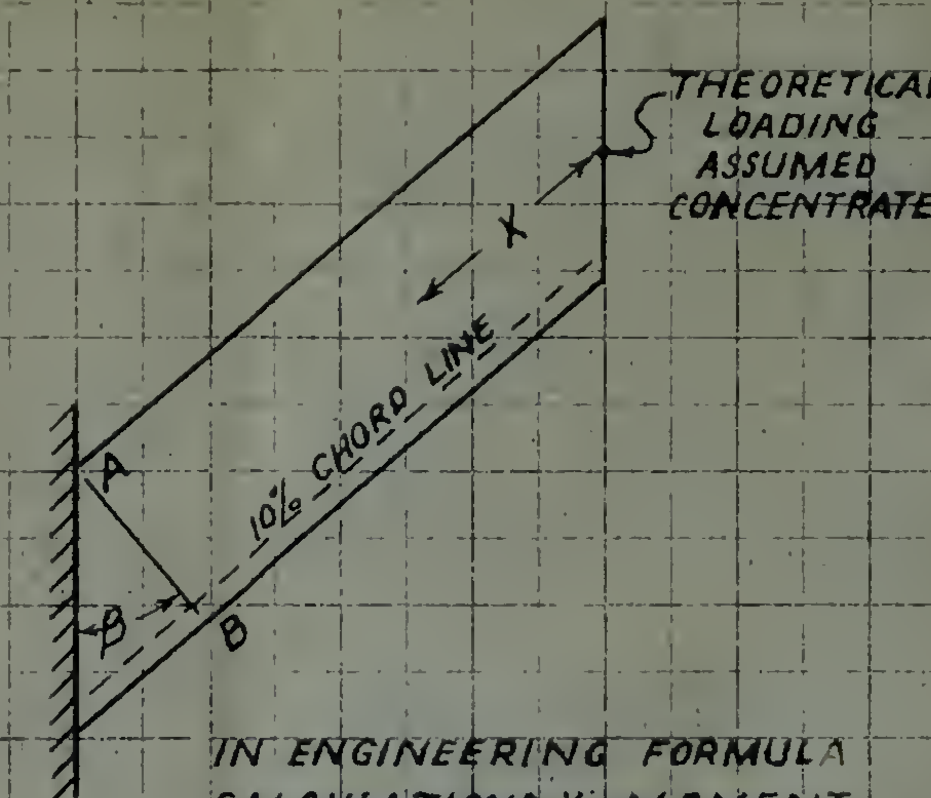
4-6-49

Figure 31(b)

STRESS VARIATION ALONG SPAN NEAR LEADING EDGE FOR VARIOUS ANGLES OF SWEEP

LOAD OF 600 LBS UNIFORMLY DISTRIBUTED OVER CHORD AT 100% SEMI SPAN





IN ENGINEERING FORMULA CALCULATIONS, X = MOMENT ARM IN FORMULA:

$$M = W X$$

PLATE TREATED AS A SIMPLE CANTILEVER BEAM WITH ROOT AT A-B

$$\sigma = \frac{My}{I}$$

— — — ENG. FORMULA

——— EXPERIMENTAL

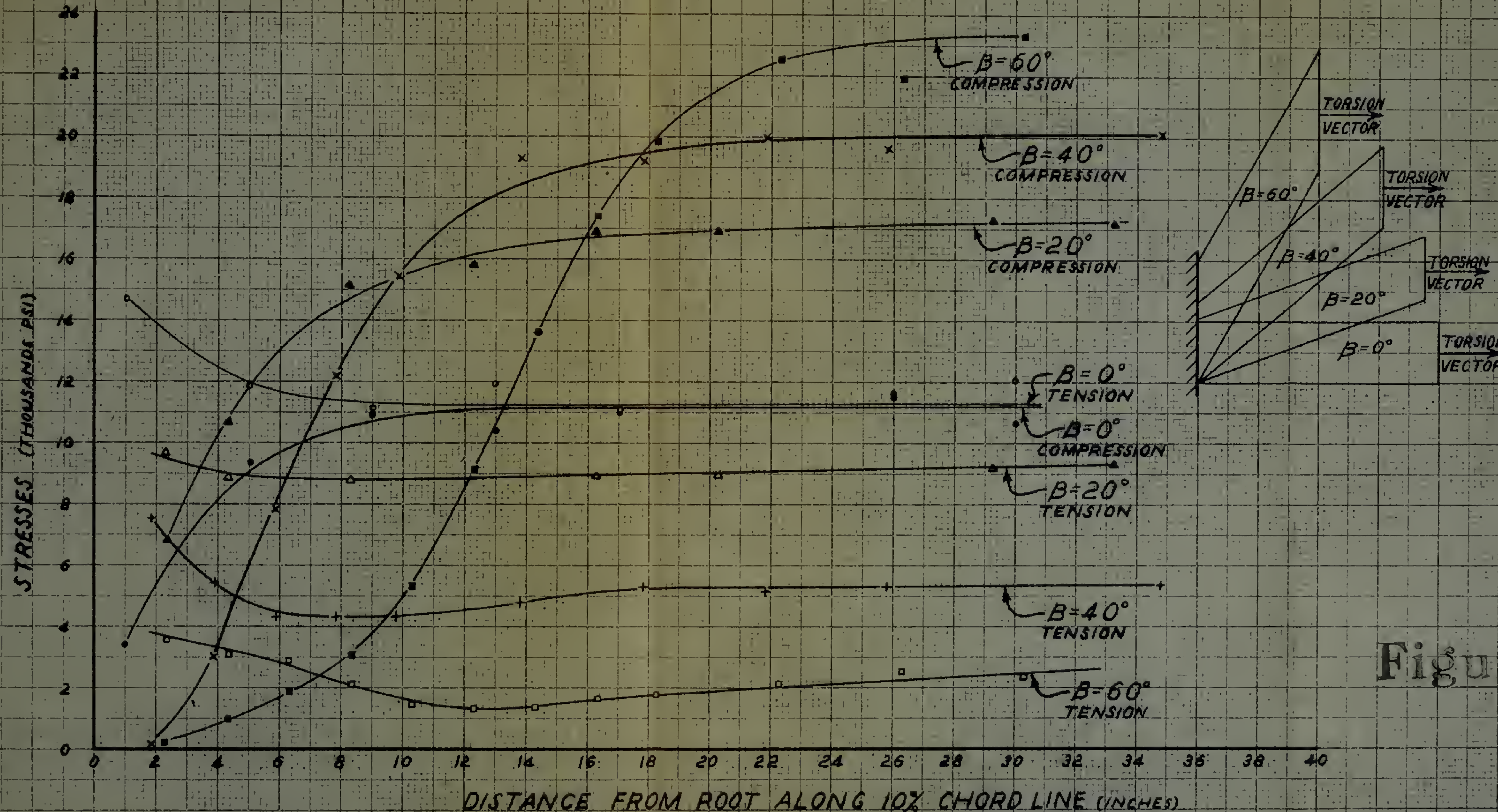


Figure 32

STRESS VARIATION ALONG SPAN NEAR LEADING EDGE FOR VARIOUS ANGLES OF SWEEP

TORSIONAL LOAD OF 45,000 IN-LBS, TORSION VECTOR PERPENDICULAR TO TIP

6-5-49

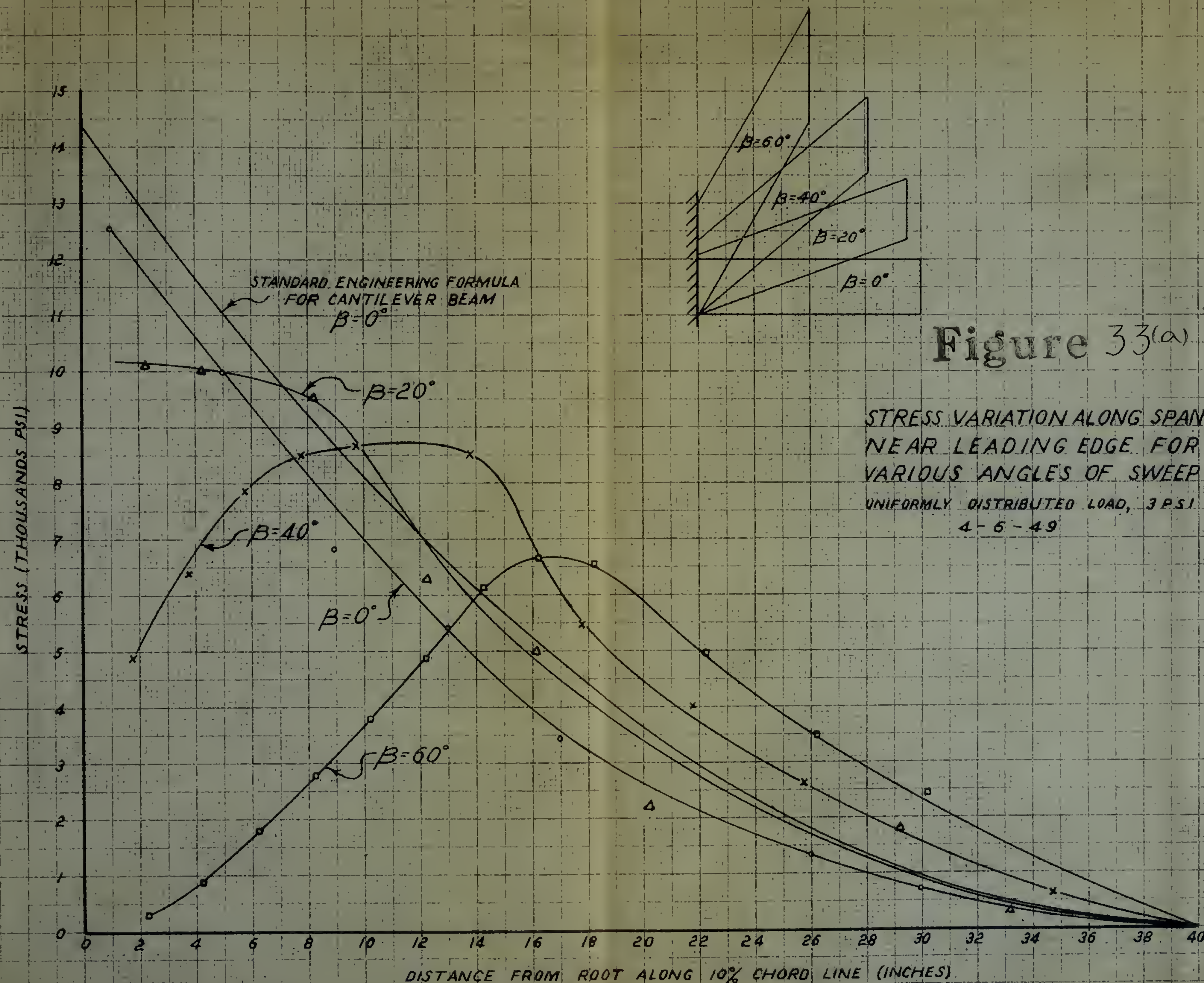


Figure 33(a)

STRESS VARIATION ALONG SPAN NEAR LEADING EDGE FOR VARIOUS ANGLES OF SWEEP

UNIFORMLY DISTRIBUTED LOAD, 3 P.S.I.

4-6-49

Figure 33(b)

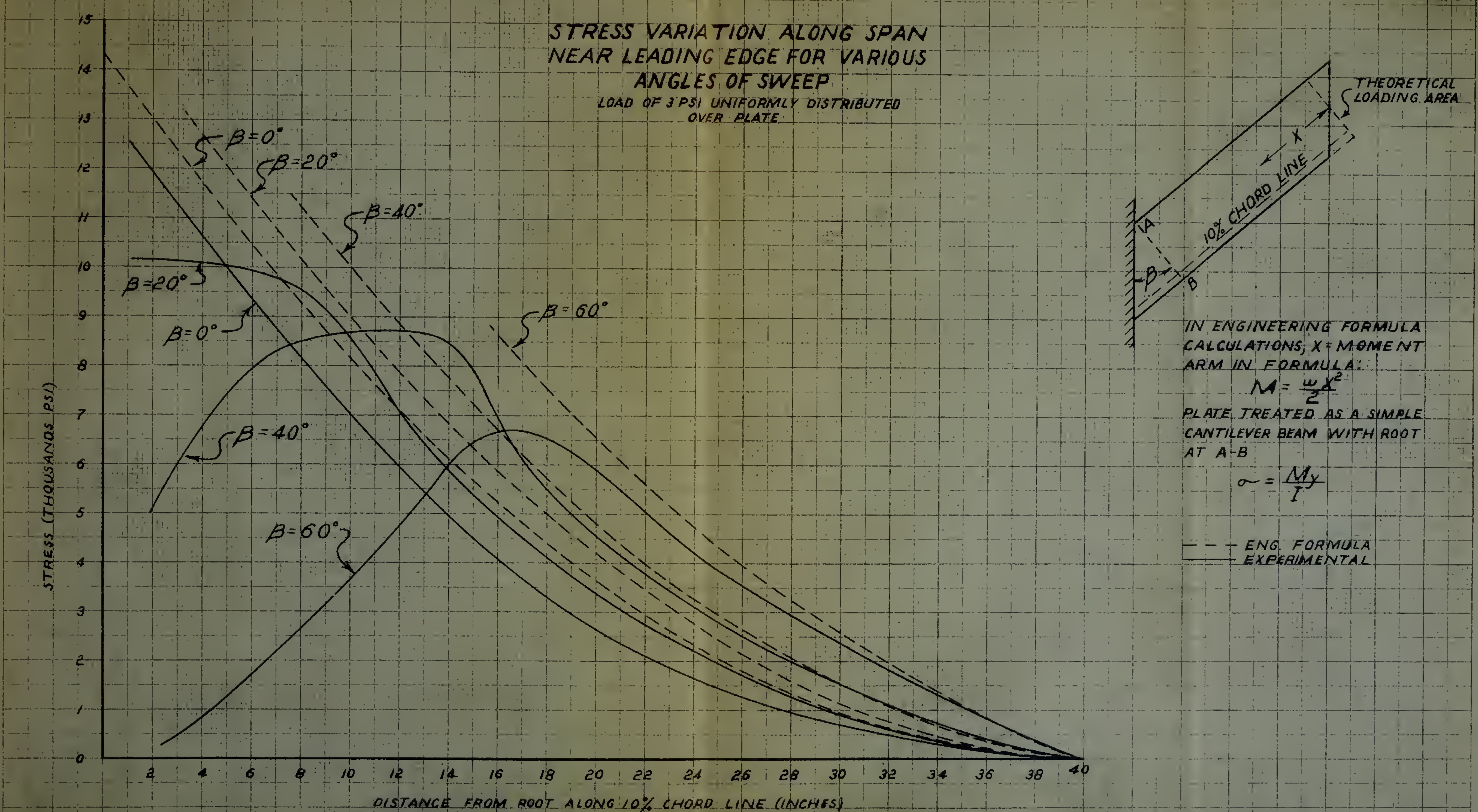
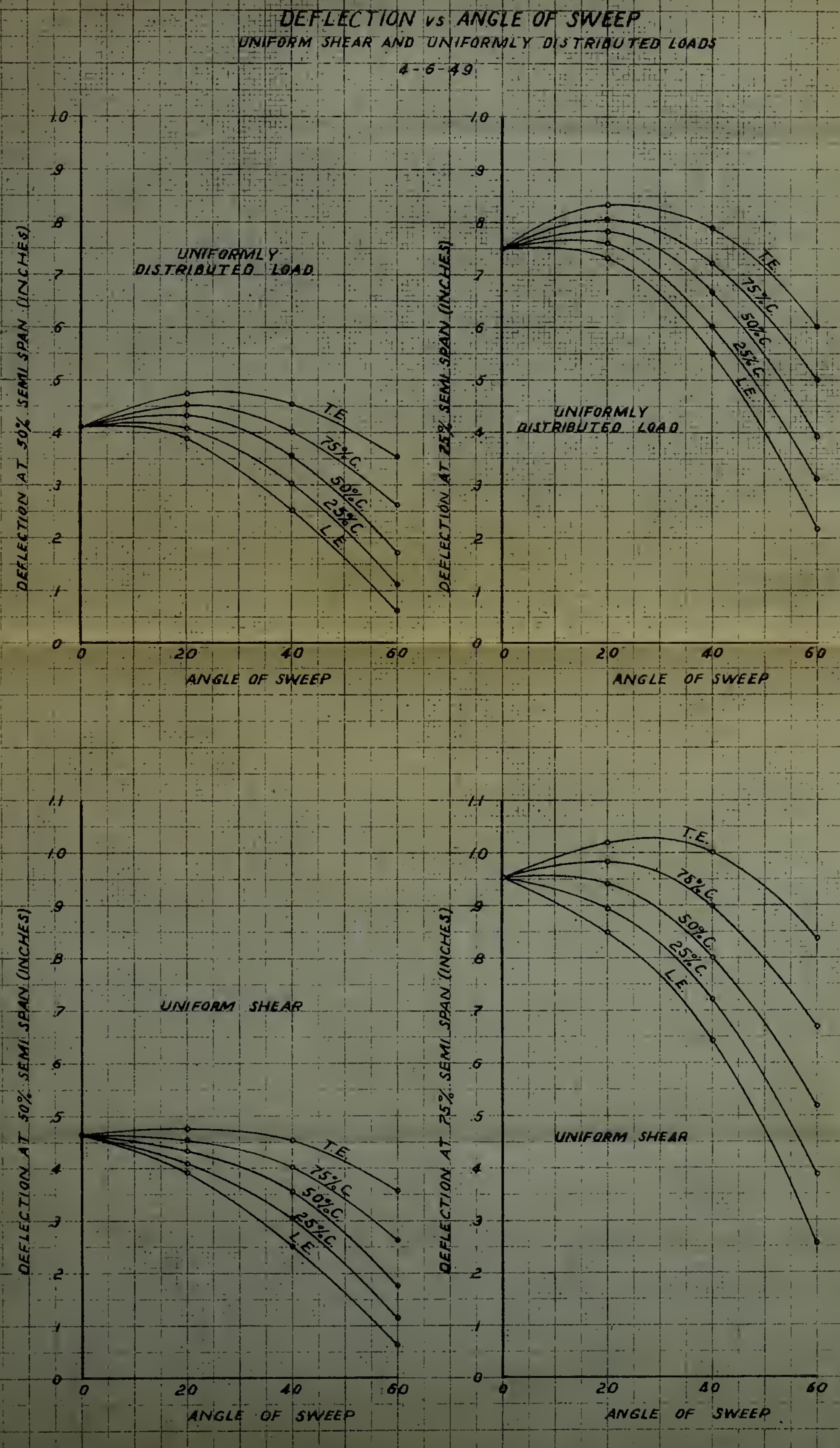

STRESS VARIATION ALONG SPAN
NEAR LEADING EDGE FOR VARIOUS
ANGLES OF SWEEP
LOAD OF 3 PSI UNIFORMLY DISTRIBUTED
OVER PLATE
β=0°
β=20°
β=40°
β=60°
THEORETICAL
LOADING AREA
X
10% CHORD LINE
A
B
β
IN ENGINEERING FORMULA
CALCULATIONS, X = MOMENT
ARM IN FORMULA:
M = wX²/2
PLATE TREATED AS A SIMPLE
CANTILEVER BEAM WITH ROOT
AT A-B
σ = My/I
ENG. FORMULA
EXPERIMENTAL
STRESS (THOUSANDS PSI)
DISTANCE FROM ROOT ALONG 10% CHORD LINE (INCHES)

Figure 34


DEFLECTION vs ANGLE OF SWEEP
UNIFORM SHEAR AND UNIFORMLY DISTRIBUTED LOADS
4-6-49
UNIFORMLY DISTRIBUTED LOAD
DEFLECTION AT 50% SEMI SPAN (INCHES)
DEFLECTION AT 25% SEMI SPAN (INCHES)
UNIFORM SHEAR
DEFLECTION AT 75% SEMI SPAN (INCHES)
ANGLE OF SWEEP
T.E.
75%C.
50%C.
25%C.
L.E.

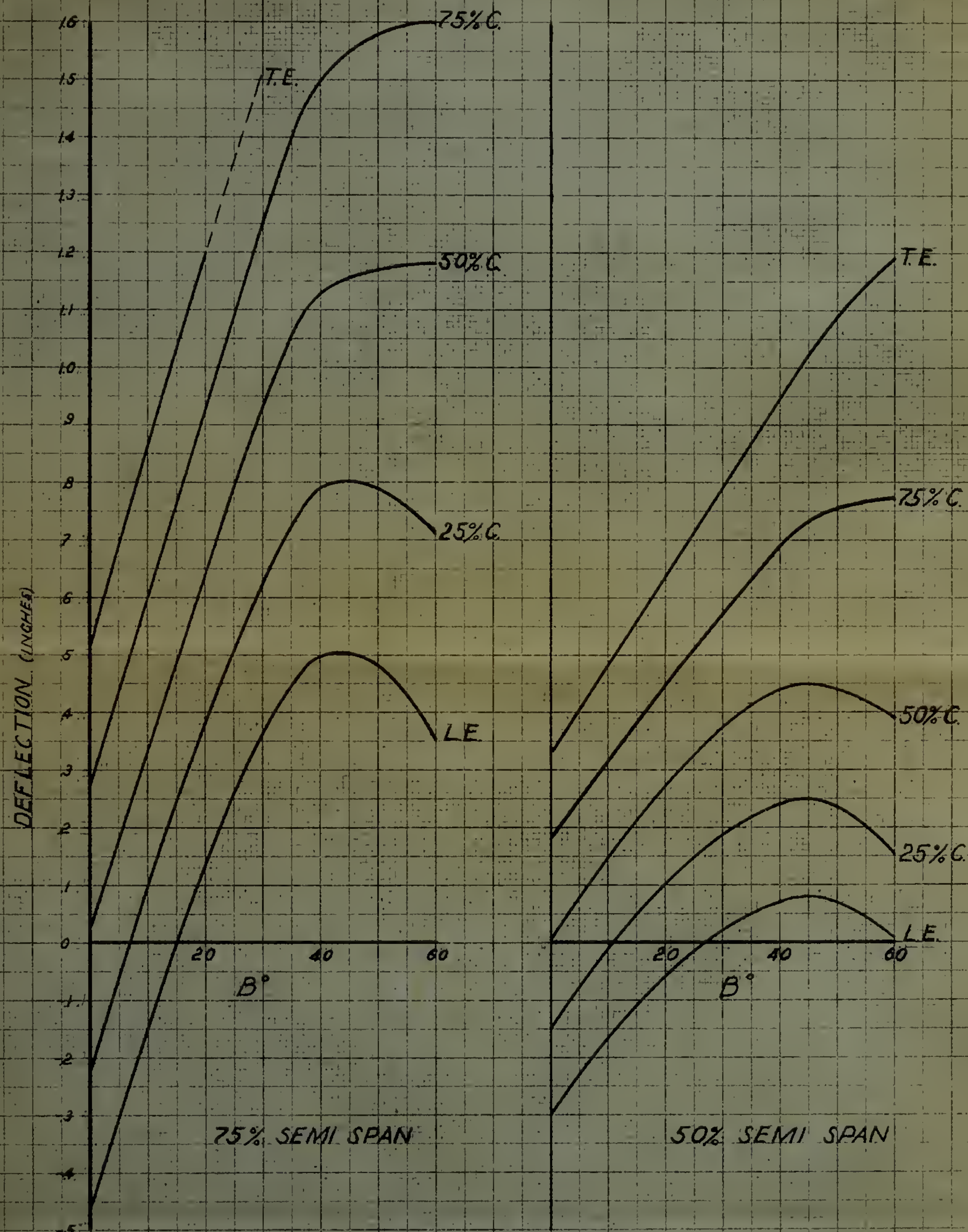


Figure 35

DEFLECTION vs ANGLE OF SWEEP
TORSION LOAD
4-6-49

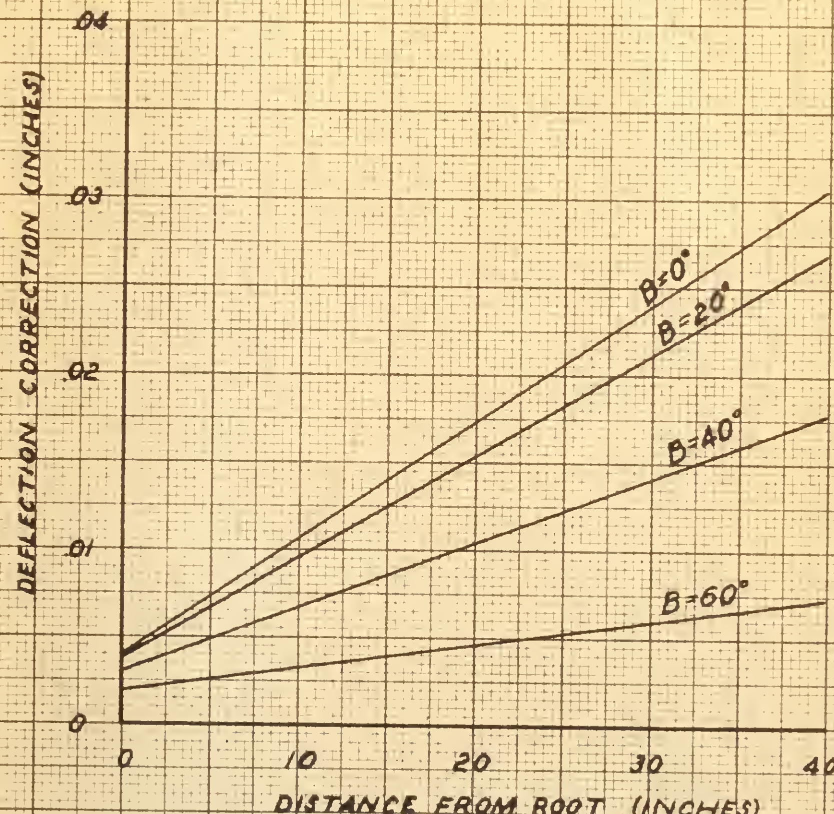


Figure 36

DEFLECTION CORRECTIONS DUE TO SAG OF SUPPORT FOR CONCENTRATED AND UNIFORM LOADS

Thesis 11598
G44 Gilkeson
An investigation of
the stresses and de-
flections of swept plates.